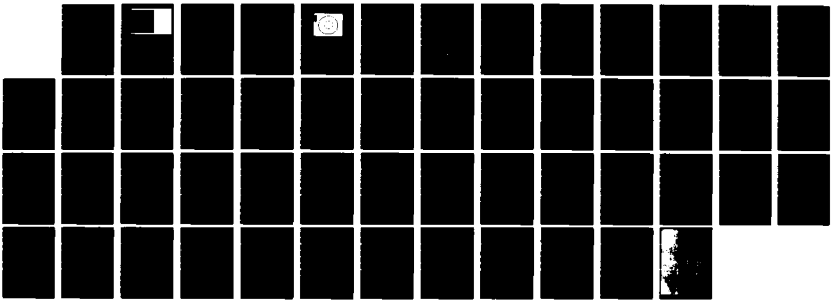
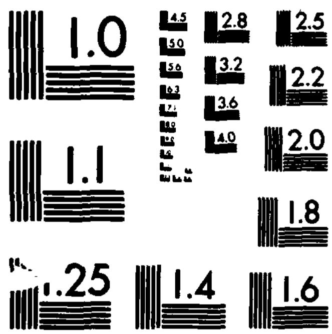
AD-A127 756 ITERATIVE NOISE ELIMINATION PRELIMINARY REPORT(U) 1/1
WISCONSIN UNIV-MADISON MATHEMATICS RESEARCH CENTER
K T SMITH JAN 83 MRC-TSR-2464 DAAG29-80-C-0041
UNCLASSIFIED F/G 20/1 NL





MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

AD A127756

MRC Technical Summary Report #2464

ITERATIVE NOISE ELIMINATION
PRELIMINARY REPORT

K. T. Smith

Mathematics Research Center
University of Wisconsin—Madison
610 Walnut Street
Madison, Wisconsin 53706

January 1983

(Received December 7, 1982)

Approved for public release
Distribution unlimited

Sponsored by

U. S. Army Research Office
P. O. Box 12211
Research Triangle Park
North Carolina 27709

National Science Foundation
Washington, D.C. 20550

Tektronix Co.
Beaverton, OR 97005

Accession For		
NTIS GRA&I		X
DTIC TAB		☐
Unannounced		☐
Justification		
By		
Distribution/		
Availability Codes		
Dist	Avail and/or Special	
A		

UNIVERSITY OF WISCONSIN-MADISON
MATHEMATICS RESEARCH CENTER

ITERATIVE NOISE ELIMINATION
Preliminary Report

K. T. Smith

Technical Summary Report #2464
January 1983

ABSTRACT

A common feature of the empirical data in applied mathematics is a low signal to noise ratio. Defined loosely as the ratio of the magnitude of essential signals to the magnitude of the worst noise, it often runs between 1/10 and 1/15, as was the case in the early work on computed tomography with digitized medical x-ray films which was the origin of this project. In such situations, straight-forward linear smoothing procedures tend to broaden the noise peaks without reducing the magnitude sufficiently.

A common redeeming feature of the noise is that the largest peaks are quite sparse, lesser peaks are more frequent but still sparse, etc. This suggests a stepwise smoothing in which only the worst peaks are removed during the early steps when there is no very good way to remove them.

Such a procedure, devised for use with medical x-ray films, is described in [1]. In the meantime it has been modified in various ways and used with other data from several sources. A detailed account of experiments with the procedure will be given elsewhere. The purpose of the present report is to describe the procedure and to show the results of a series of tests with data from a computed tomography x-ray scan of a defective battery. The scan was taken with a developmental version of an industrial scanner at the General Electric Research and Development Center by H. J. Scudder.

AMS(MOS) Subject Classification: 41-04, 65D10

Key Words: Noise, Computed tomography

Work Unit Number 1 - Applied Analysis

Sponsored by the U. S. Army under Contract No. DAAG29-80-C-0041. This material is based upon work supported by the National Science Foundation under Grant No. MCS-8101586, and by the Tektronix Co. through provision of graphics equipment.

ITERATIVE NOISE ELIMINATION

Preliminary Report

by

K. T. Smith

1. Introduction.

A common feature of the empirical data in applied mathematics is a low signal to noise ratio. Defined loosely as the ratio of the magnitude of essential signals to the magnitude of the worst noise, it often runs between 1/10 and 1/15, as was the case in the early work on computed tomography with digitized medical x-ray films which was the origin of this project. In a recent coffee conversation on small parameter asymptotics in turbulence theory, P. G. Saffman remarked that the only true small parameter in turbulence theory is the signal to noise ratio. In such situations, straightforward linear smoothing procedures tend to broaden the noise peaks without reducing the magnitude sufficiently.

A common redeeming feature of the noise is that the largest peaks are quite sparse, lesser peaks are more frequent but still sparse, etc. This suggests a stepwise smoothing in which only the worst peaks are removed during the early steps when there is no very good way to remove them.

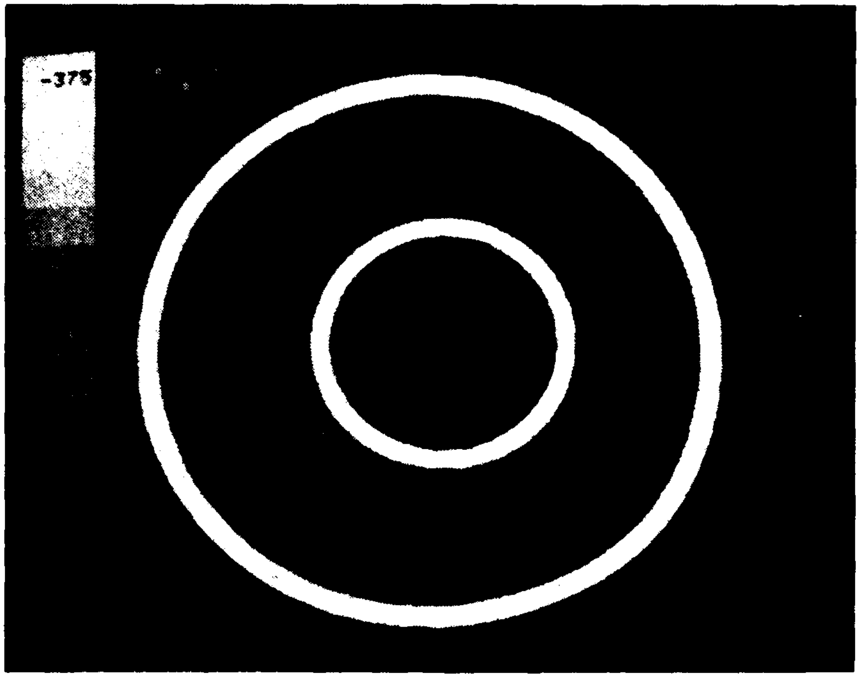
Such a procedure, devised for use with medical x-ray films, is described in [1]. In the meantime it has been modified in various ways and used with other data from several sources. A detailed account of experiments with the procedure will be given elsewhere. The purpose of the present report is to describe the procedure and to show the results of a series of tests with data from a computed tomography x-ray scan of a defective battery. The scan was taken with a developmental version of an industrial scanner at the General Electric Research and Development Center by H. J. Scudder.

Table of Contents

Sponsored by the U. S. Army under Contract No. DAAG29-80-C-0041. This material is based upon work supported by the National Science Foundation under Grant No. MCS-8101586, and by the Tektronix Co. through provision of graphics equipment.

2. The battery and battery phantom.

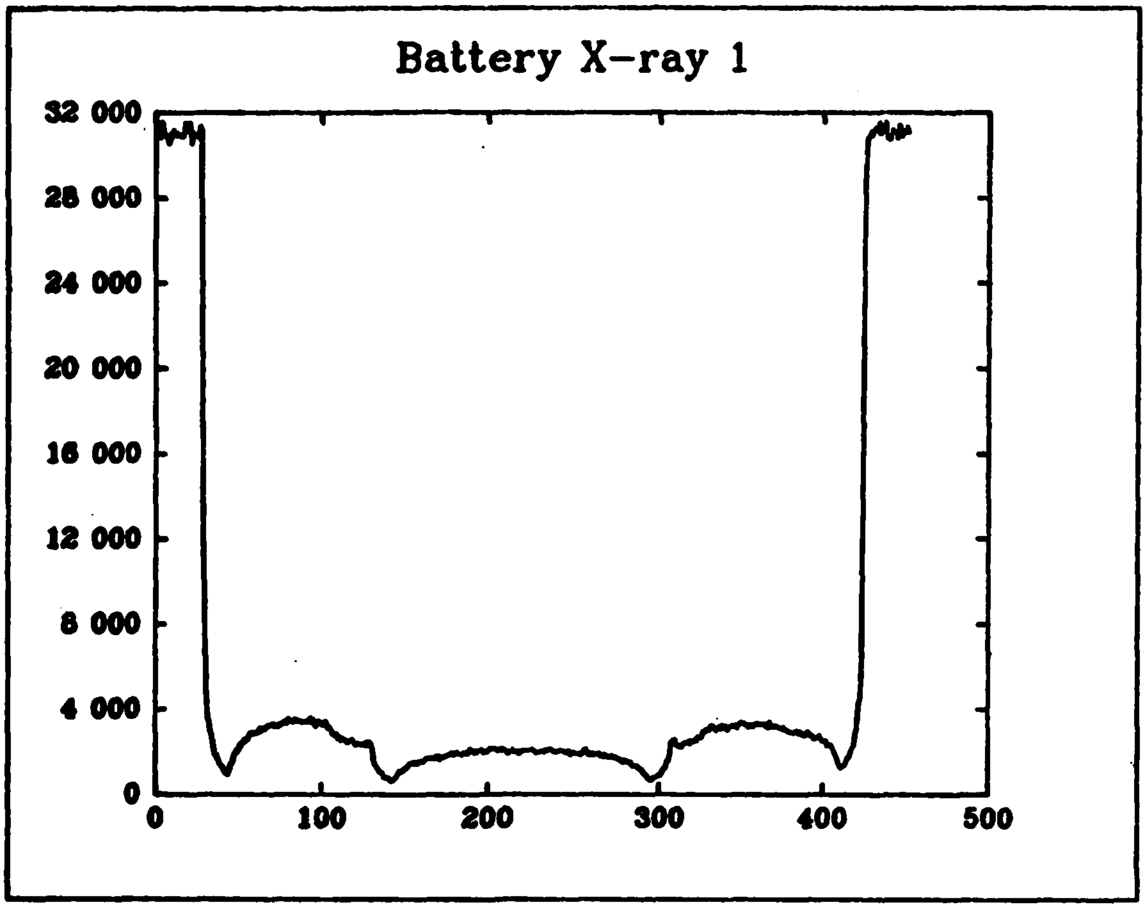
The battery was made of ceramic material with outer and inner steel shells. A CT reconstruction is shown below.

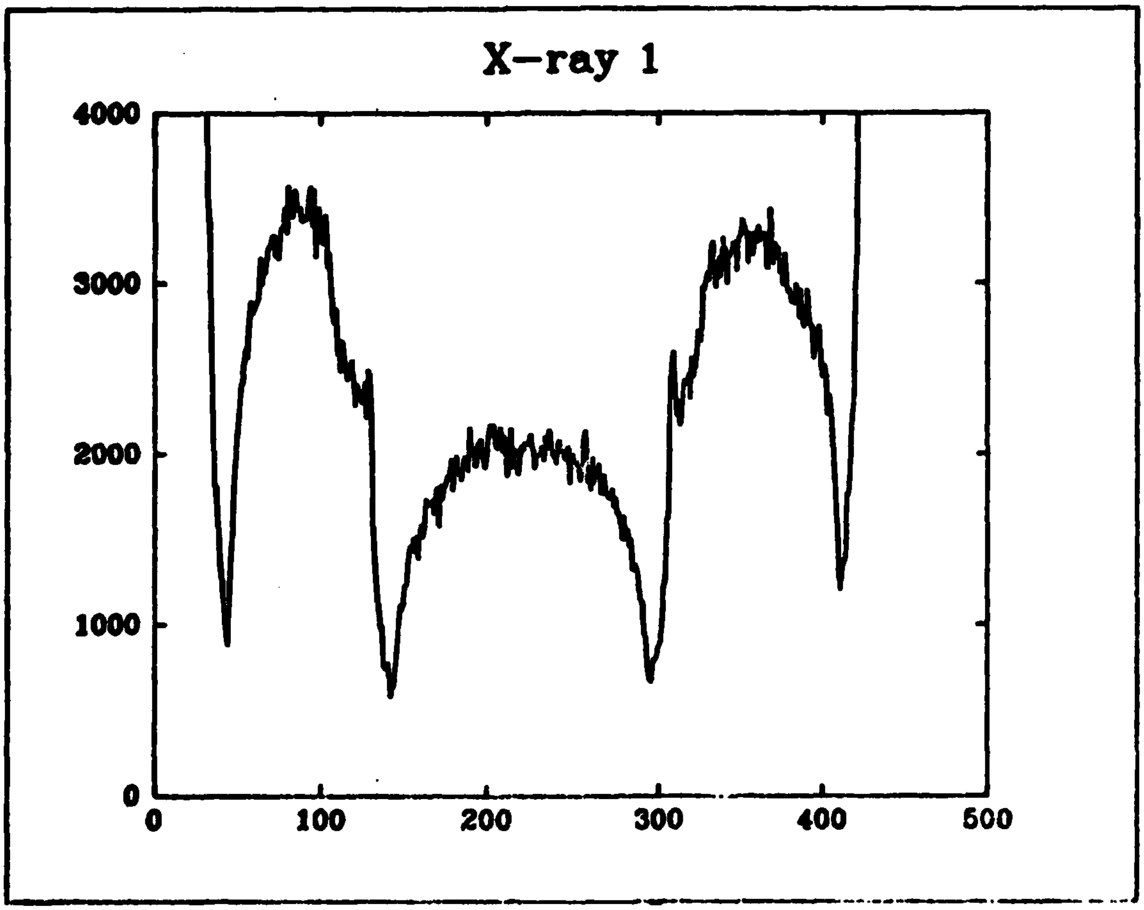


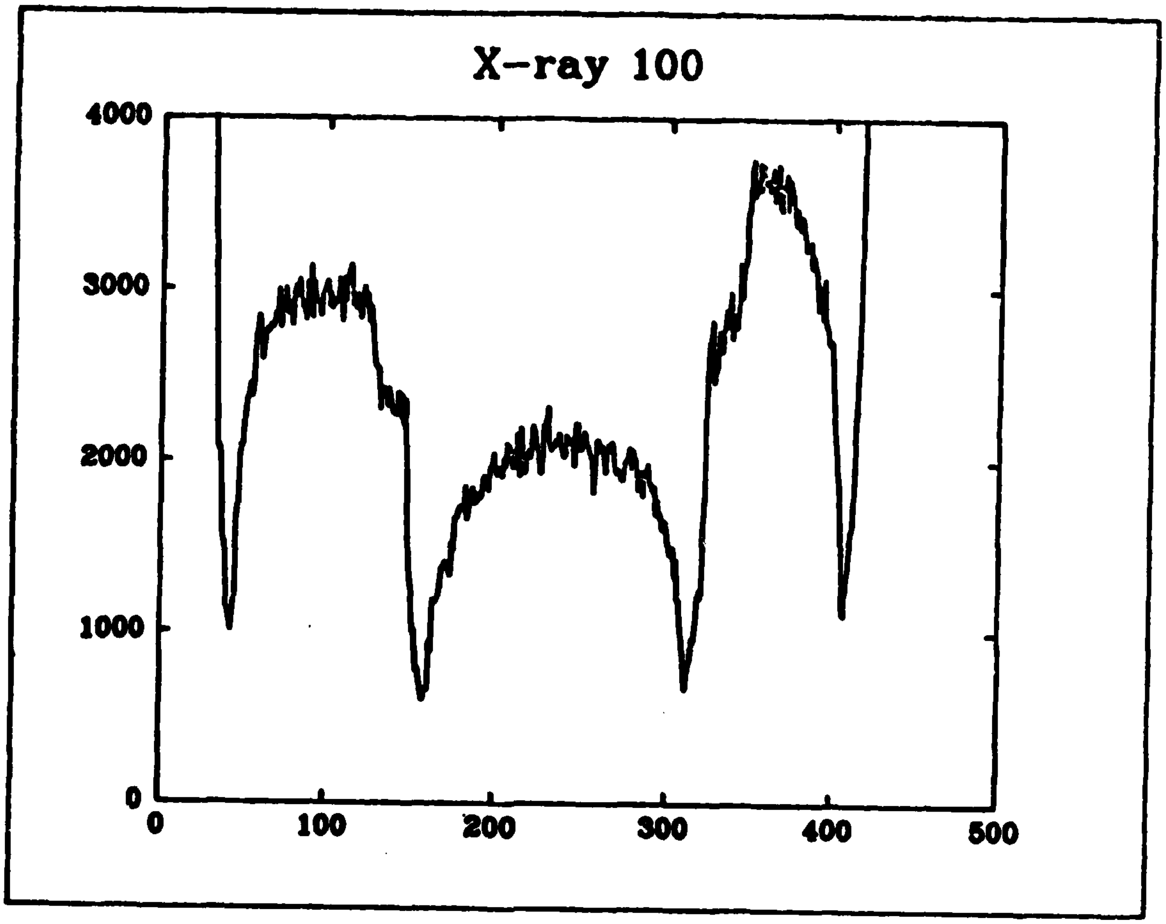
Battery Cross Section

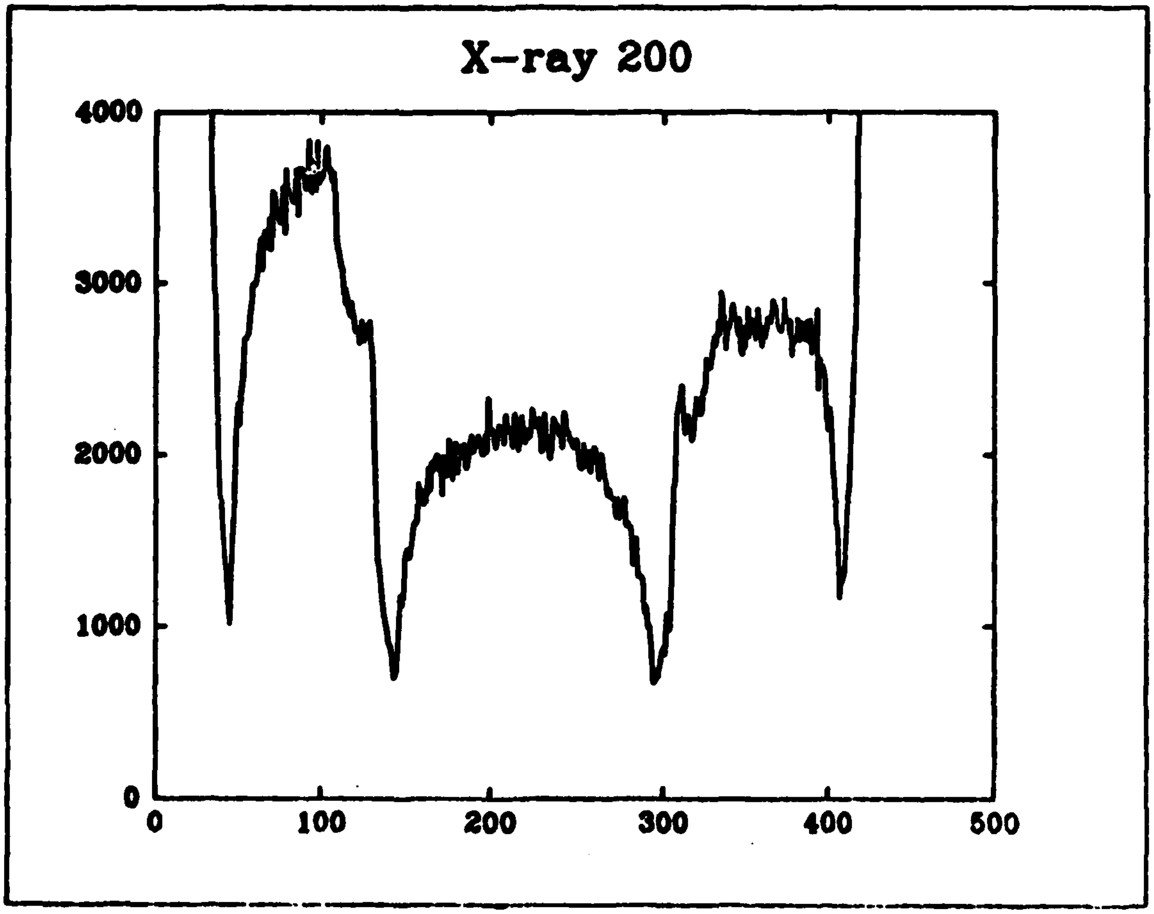
The scan data consisted of 256 radiographs (referred to hereafter as x-rays), each one digitized at 452 points. Sample graphs are shown below. The ceramic interior is rather dense, and the steel shells are very dense. Consequently, although the data vary over the range 0-32000, the variations of interest lie in the range 0-4000. In all graphs after the first the data are cut at 4000 so that variations of interest can be seen.

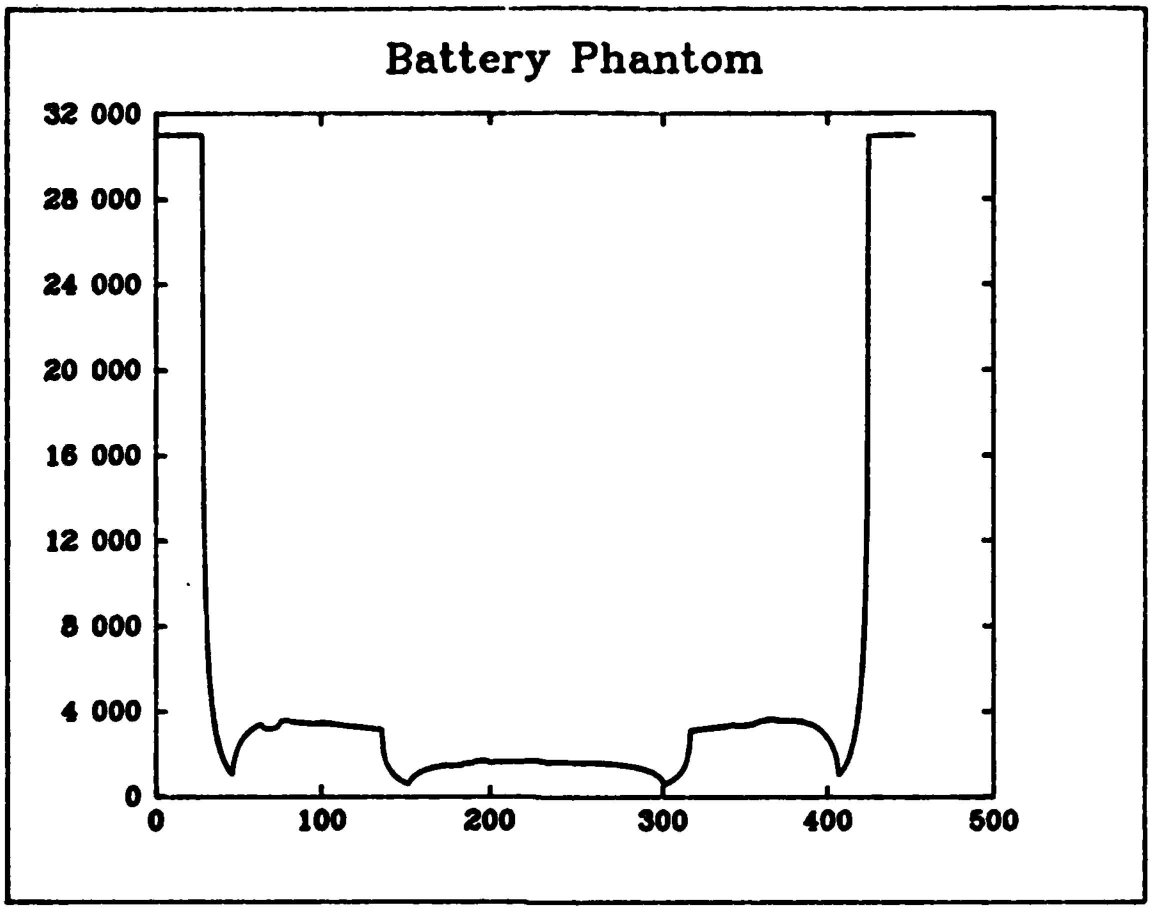
To test the noise elimination procedure, a battery phantom was created mathematically. The phantom has the gross features of the battery, along with explicitly known small variations. Noise was extracted from the battery x-rays and added to the phantom to produce noisy phantoms for the noise elimination procedure, the objective being to test how well the procedure could reproduce the correct phantom with its sharp peaks and small variations. Graphs of the original phantom, two noisy phantoms, and their smoothed counterparts are shown below. For purposes of comparison, the original phantom is overlaid on the reconstructions.

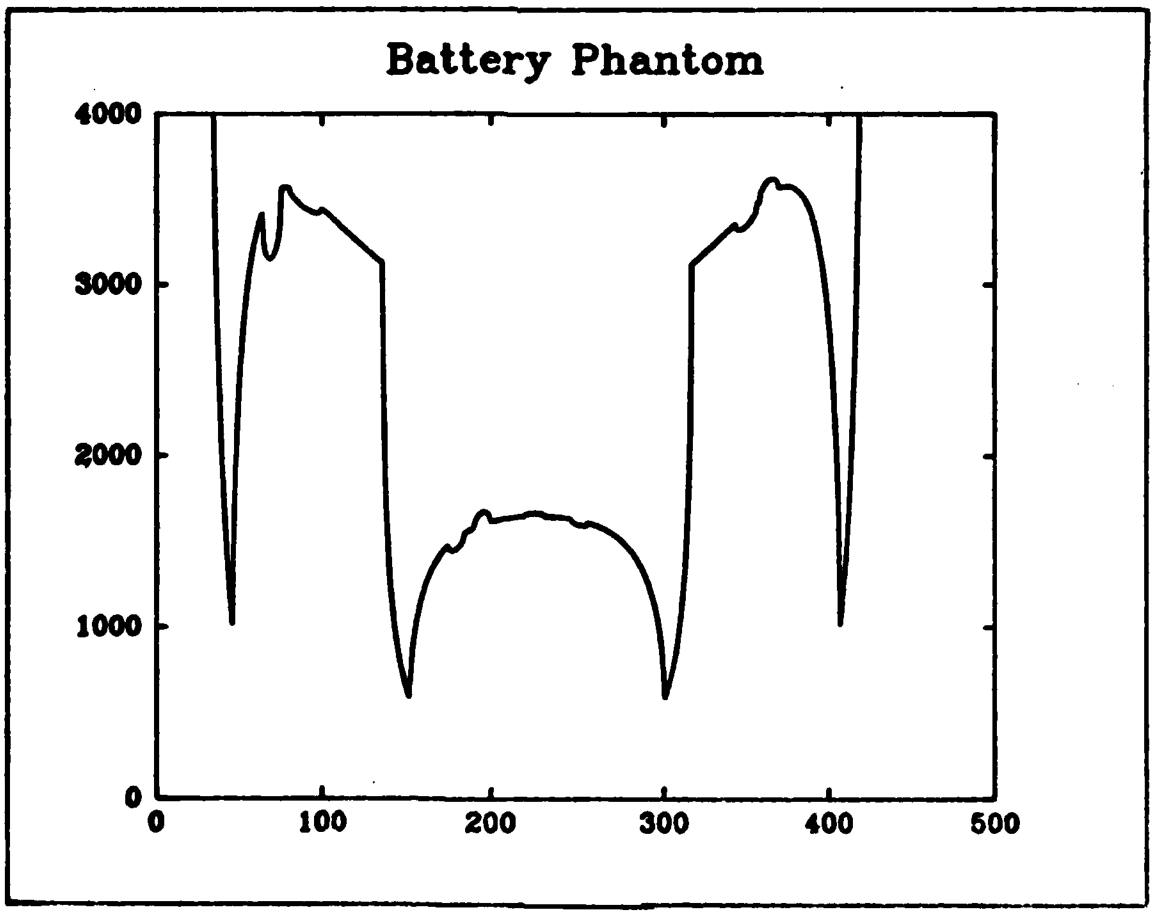

Battery X-ray 1
32 000
28 000
24 000
20 000
16 000
12 000
8 000
4 000
0
0
100
200
300
400
500

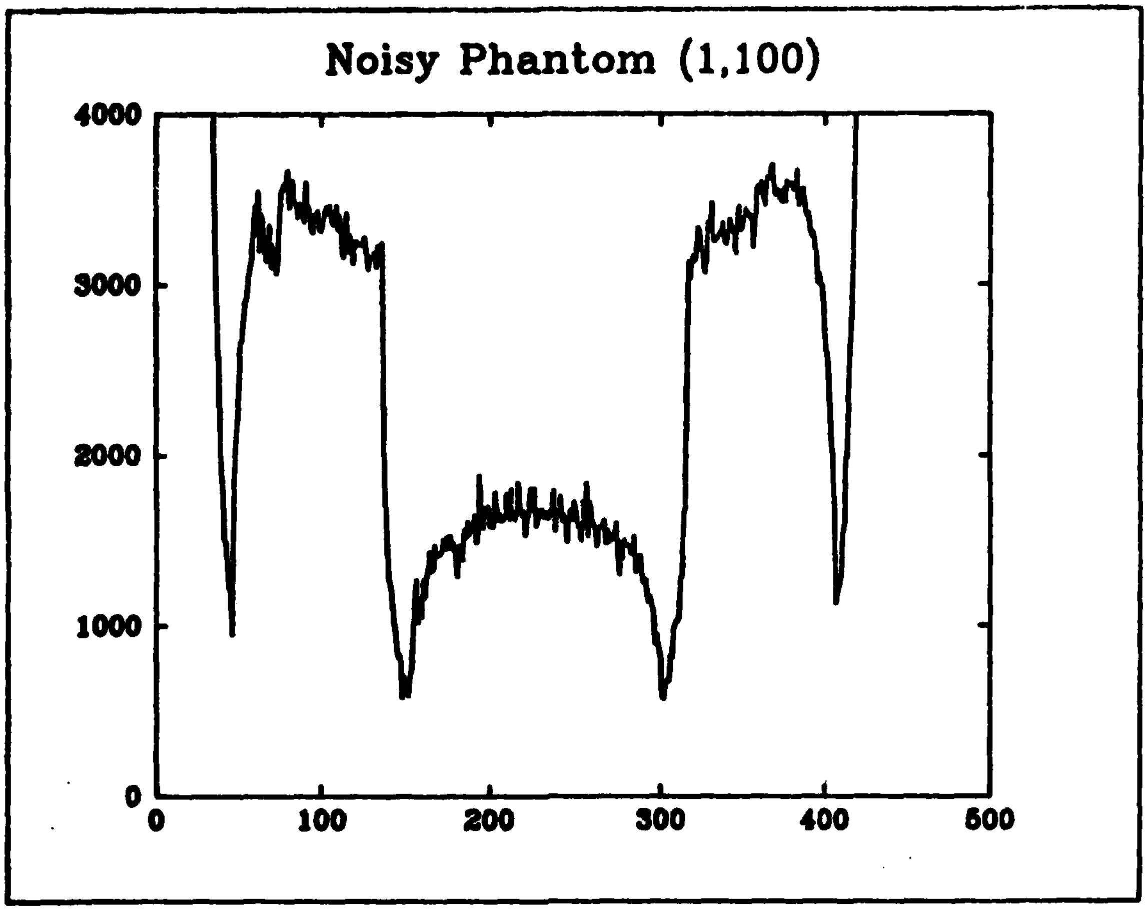

X-ray 1
4000
3000
2000
1000
0
0
100
200
300
400
500

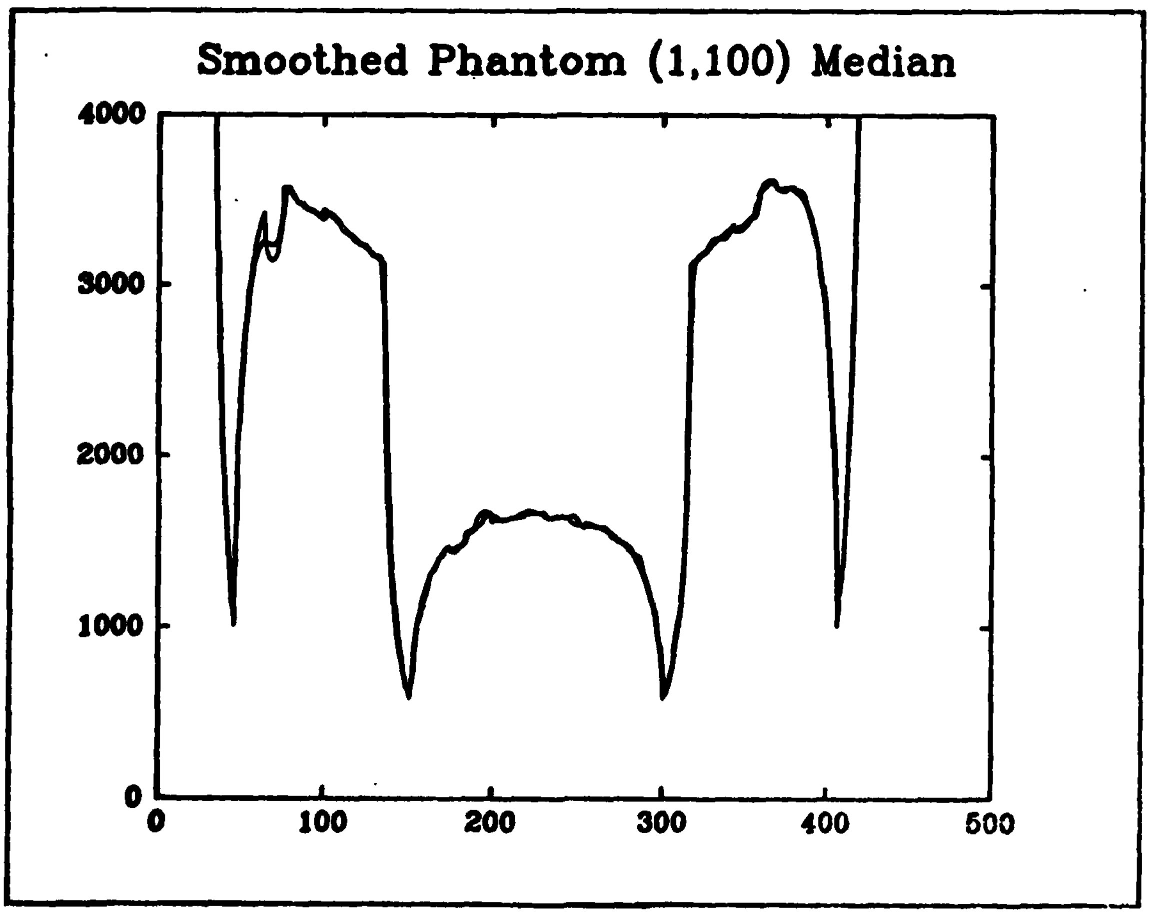

X-ray 100
4000
3000
2000
1000
0
0
100
200
300
400
500

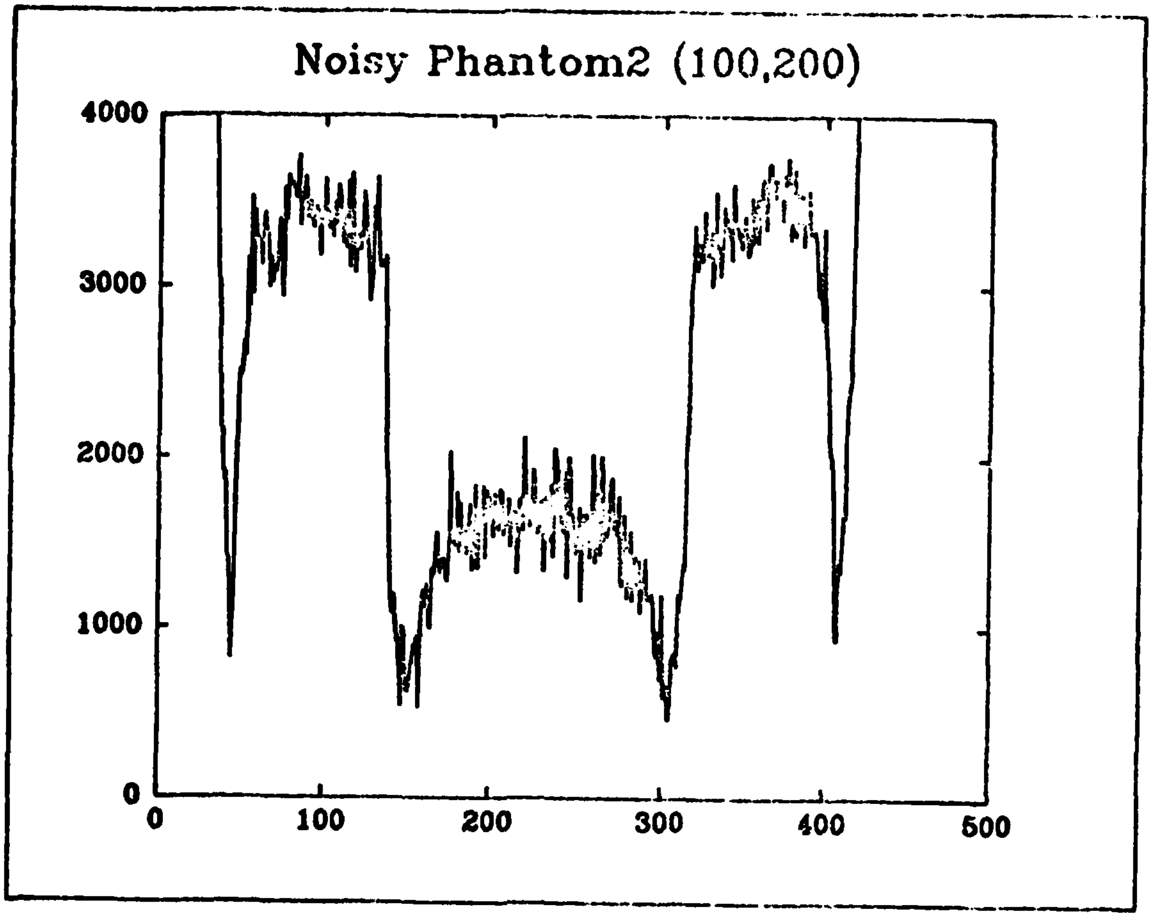

X-ray 200
4000
3000
2000
1000
0
0
100
200
300
400
500

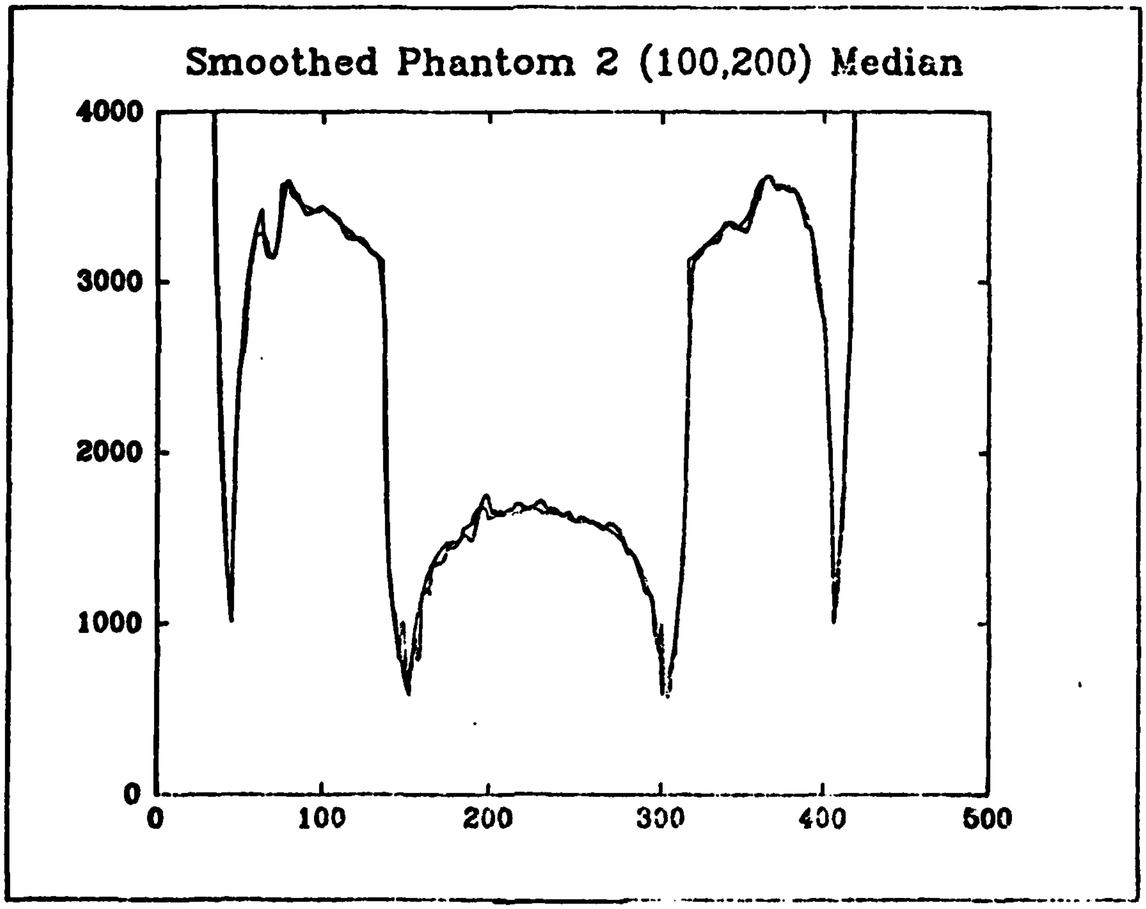

Battery Phantom
32 000
28 000
24 000
20 000
16 000
12 000
8 000
4 000
0
0
100
200
300
400
500


Battery Phantom
4000
3000
2000
1000
0
0
100
200
300
400
500


Noisy Phantom (1,100)
4000
3000
2000
1000
0
0
100
200
300
400
500


Smoothed Phantom (1,100) Median
4000
3000
2000
1000
0
0
100
200
300
400
500


Noisy Phantom2 (100,200)
4000
3000
2000
1000
0
0
100
200
300
400
500


Smoothed Phantom 2 (100,200) Median
4000
3000
2000
1000
0
0
100
200
300
400
500

3. A description of the procedure.

The iterative procedure involves K steps, K interval lengths $L_1,\ldots,L_K$, and K percentage parameters $p_1,\ldots,p_K$. In the present examples these parameters have the following values:

$$K = 7, \quad L = (8,8,6,6,4,3,3), \quad p = (.02,.02,.04,.08,.12,.24,1)$$

The data are given as a numerical sequence D of length n. At step k, corrections are made only at the p_k*n points where the noise is worst, and the corrected data at the end of step k are denoted by D_k. Thus, $D_0 = D$.

At the beginning of step k+1 there is computed a sequence N_k which gives a measure of the noise in D_k at each index i. From the sequence N_k there is derived the number M_k such that the condition

$$(3.1) \qquad N_k(i) \geq M_k$$

is satisfied at p_k*n indices i. Corrections are then made at the points satisfying this condition. What remains is to specify how the noise N_k and the correction D_{k+1} are determined - points that remain somewhat up in the air.

A basic assumption of the method is that the true, noiseless data is smooth, except perhaps for occasional jump discontinuities. In line with this assumption, "average" values are computed: a left average $AL_k(i)$ of D_k over the interval of length L_k with right end point at i, a corresponding right average $AR_k(i)$, and a total average $A_k(i)$ over the inteval of length $2L_k+1$ and center i. The noise $N_k(i)$ is then the minimum of $|D_k(i)-AL_k(i)|$, $|D_k(i)-AR_k(i)|$, and $|D_k(i)-A_k(i)|$. When a correction is necessary, according to the criterion (3.1), $D_{k+1}(i)$ is something similar to $A_k(i)$.

The "averages" can be formed in various ways, three of which are examined in this report. In the first AL, AR, and A are ordinary averages, while $D_{k+1}(i)$ is an ordinary average over points that do not differ too much from $A_k(i)$. In the second AL, AR, and A are ordinary averages again, but $D_{k+1}(i)$ is roughly the median of D_k over the symmetric interval. In the third the "averages" are all essentially medians. The results in the second case seem better than those in the first, while the results in the third case seem better than those in the second. On the other hand, the averages are faster computationally, though this question has not received much consideration. It is possible that the procedure of cases 2 and 3 should be followed in the earlier steps, along with more sophisticated ones in the later steps.

The battery data considered in this report do not satisfy the smoothness condition that is basic to the method. The steel shells produce very large sharp peaks. In such a case, the noisiest points (by the criterion above) all lie around the peaks, and the criterion gives a poor measure of true noise. All early corrections are made near the peaks so that the true peaks are eliminated, while the true noise is left largely unmolested. In order to avoid this, the peaks are cut off, the resulting data is run through the procedure, and the peaks are then put back, slightly smoothed. Around the peaks this process leaves some noise, which is almost invisible in Smoothed Phantom (1,100) but is clearly visible in Smoothed Phantom 2 (100,200), where the initial noise is much larger. It must be expected that different kinds of data will exhibit different problems that can be dealt with only on the basis of special information about the data and the use to which it will be put. For example, the systematic errors around the peaks, which cause no problem in the visual examination of Smoothed Phantom (1,100), might be very damaging when put into CT reconstruction formulas.

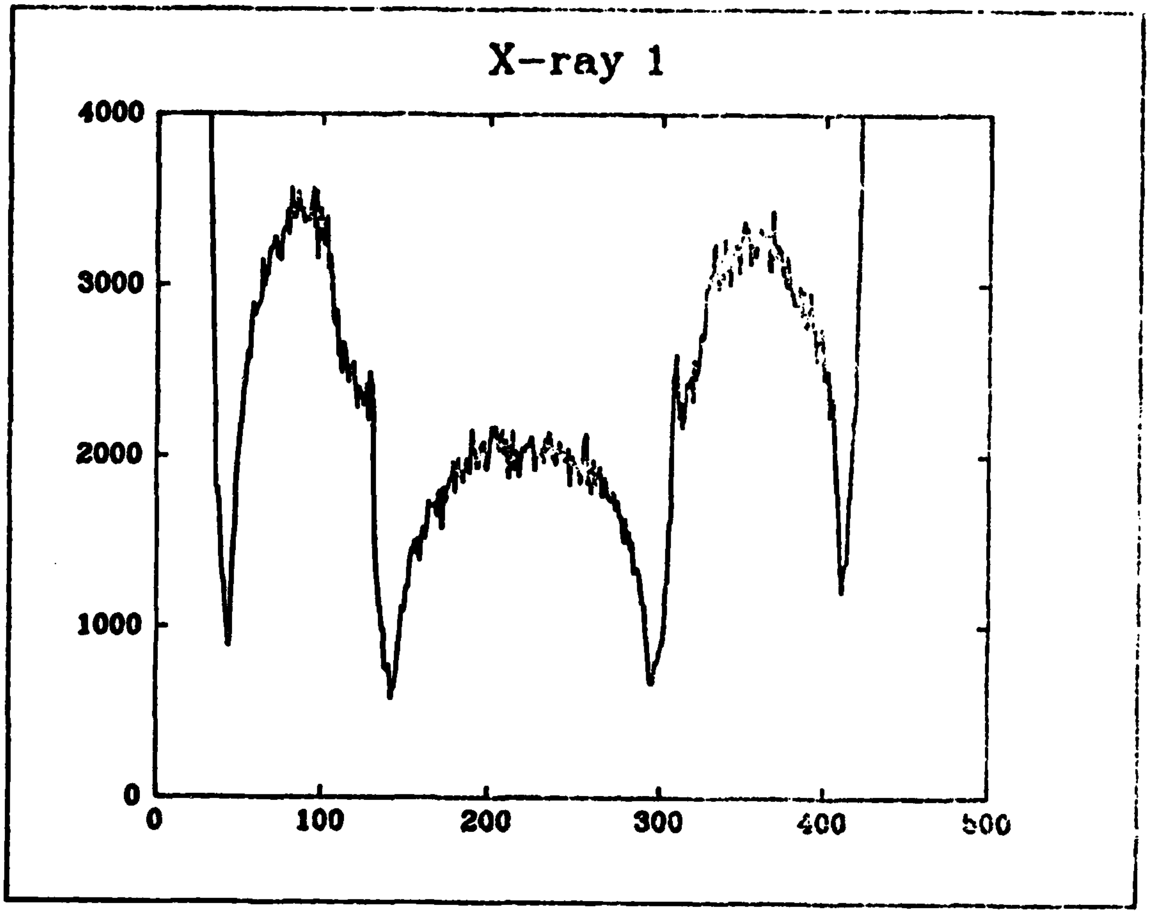
4. Smoothing with averages.

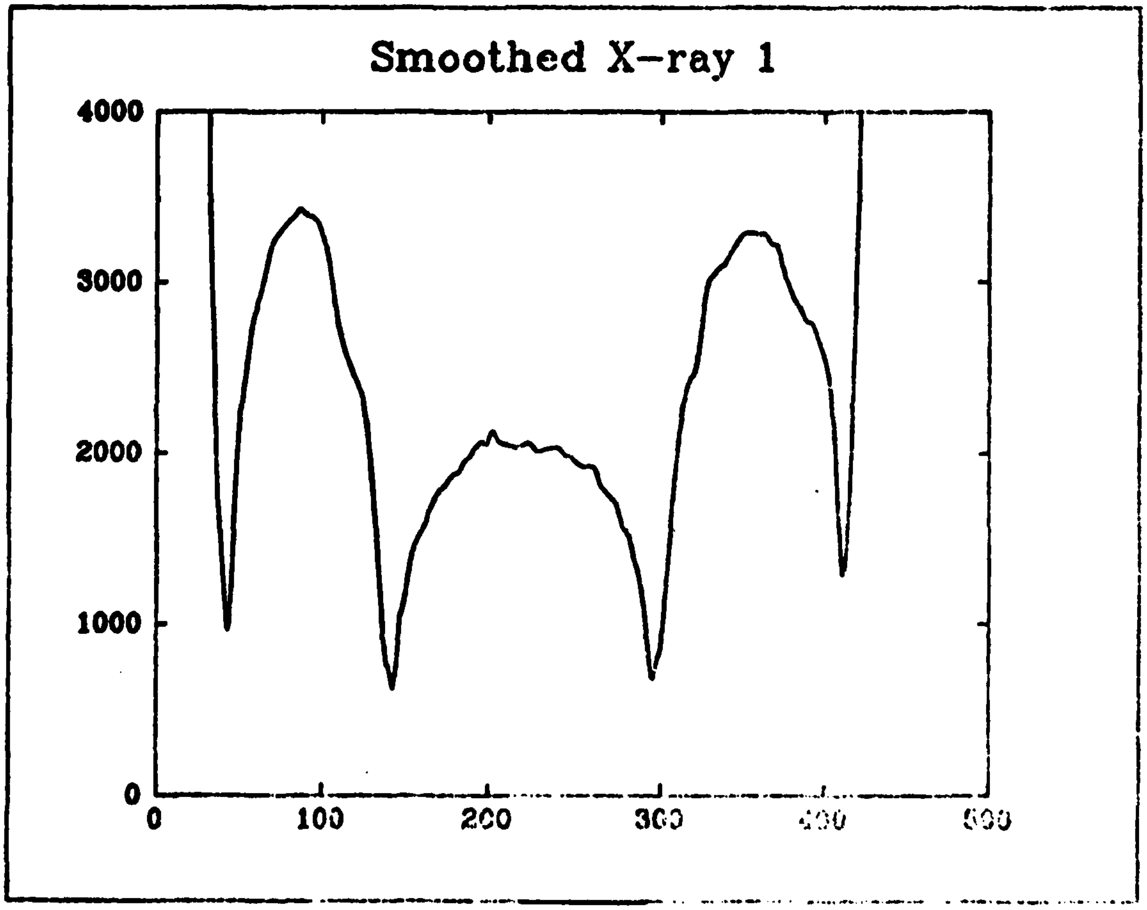
In this section the smoothing is done with averages: $AL_k(i)$ is the average of D_k over the interval of length L_k with right endpoint at i, $AR_k(i)$ is the corresponding right hand average, and $A_k(i)$ is the average of D_k over the interval of length $2L_k+1$ with center at i. $D_{k+1}(i)$ is the average of D_k over those points in the latter interval satisfying $N_k(j) < M_k'$, with M_k' chosen so that overall 80% of the points satisfy the condition. The first graphs below show x-rays 1, 100, and 200 along with their smoothings.

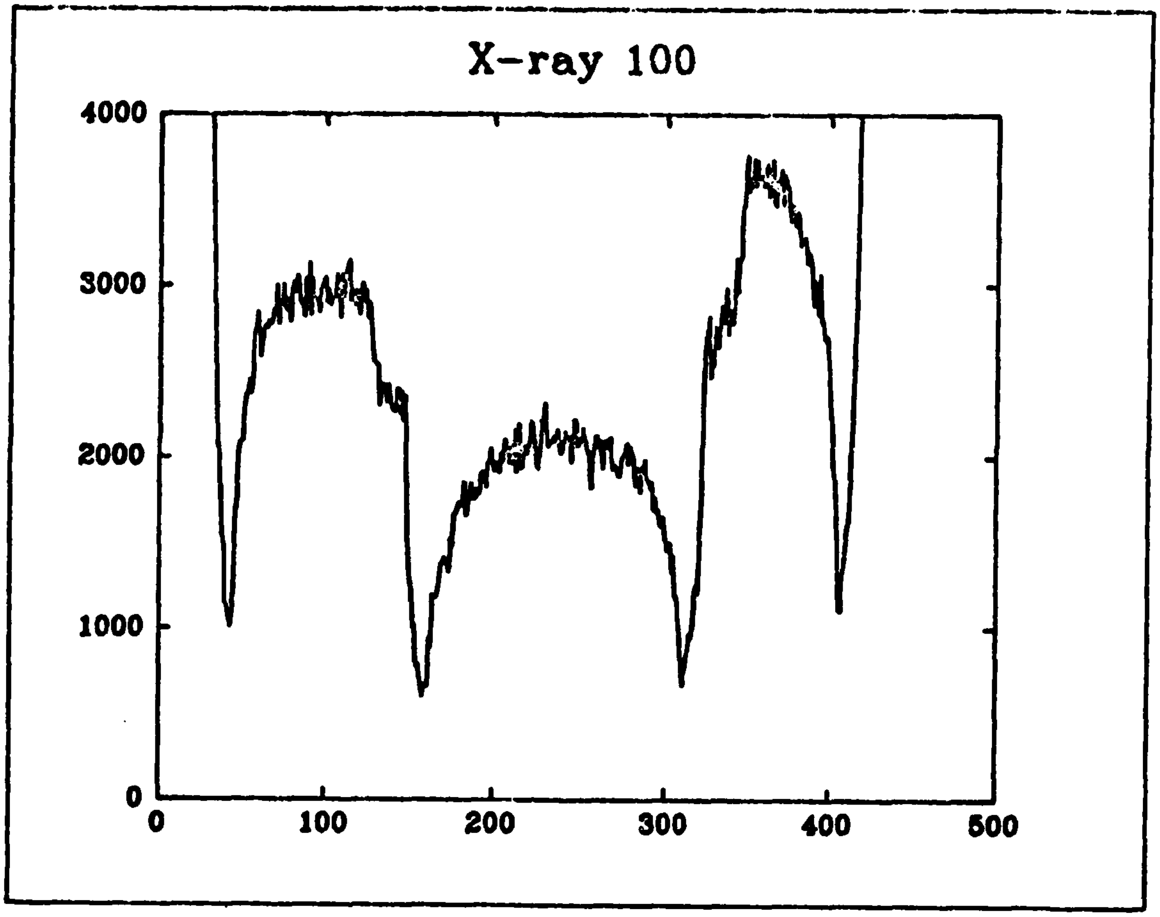
Since the battery is not explicitly known, the graphs give no measure of the accuracy of the smoothing procedure. However, the differences between the original and smoothed versions serve to provide noise to add to the known phantom. These differences are shown in the next set of graphs. Since the smoothed versions are smooth, and the noise is highly oscillatory, the differences include the noise, but they also include artifacts of the procedure. In particular, the large noise peaks at the sites of the large steel peaks of the battery are artifacts.

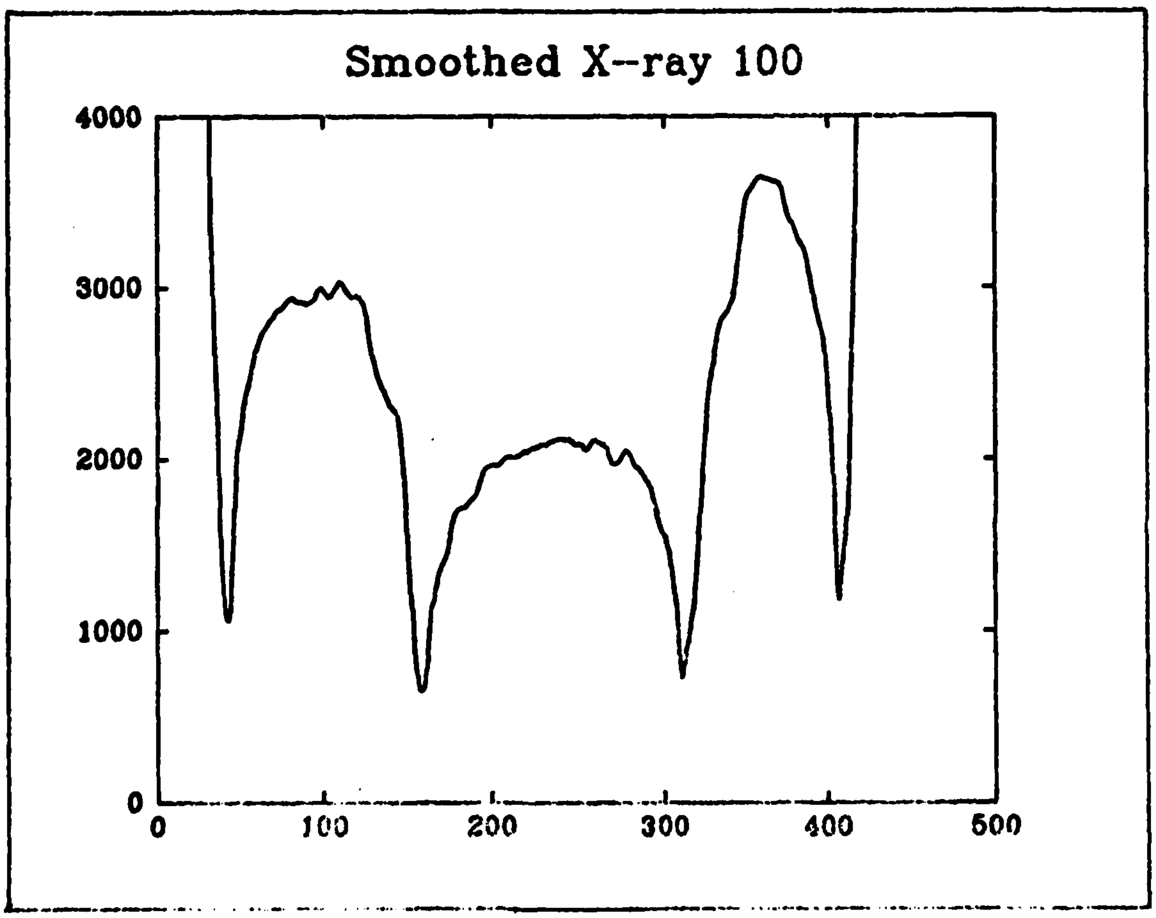
The noise added to the phantom comes from these differences with intervals around the steel peaks omitted. Specifically, the Noise (1,100) comes partly from x-ray 1 and partly from x-ray 100, while the Noise2 (100,200) comes partly from x-ray 100 and partly from x-ray 200, but is the double of the differences. (In many problems the noise has larger magnitude than it does here.)

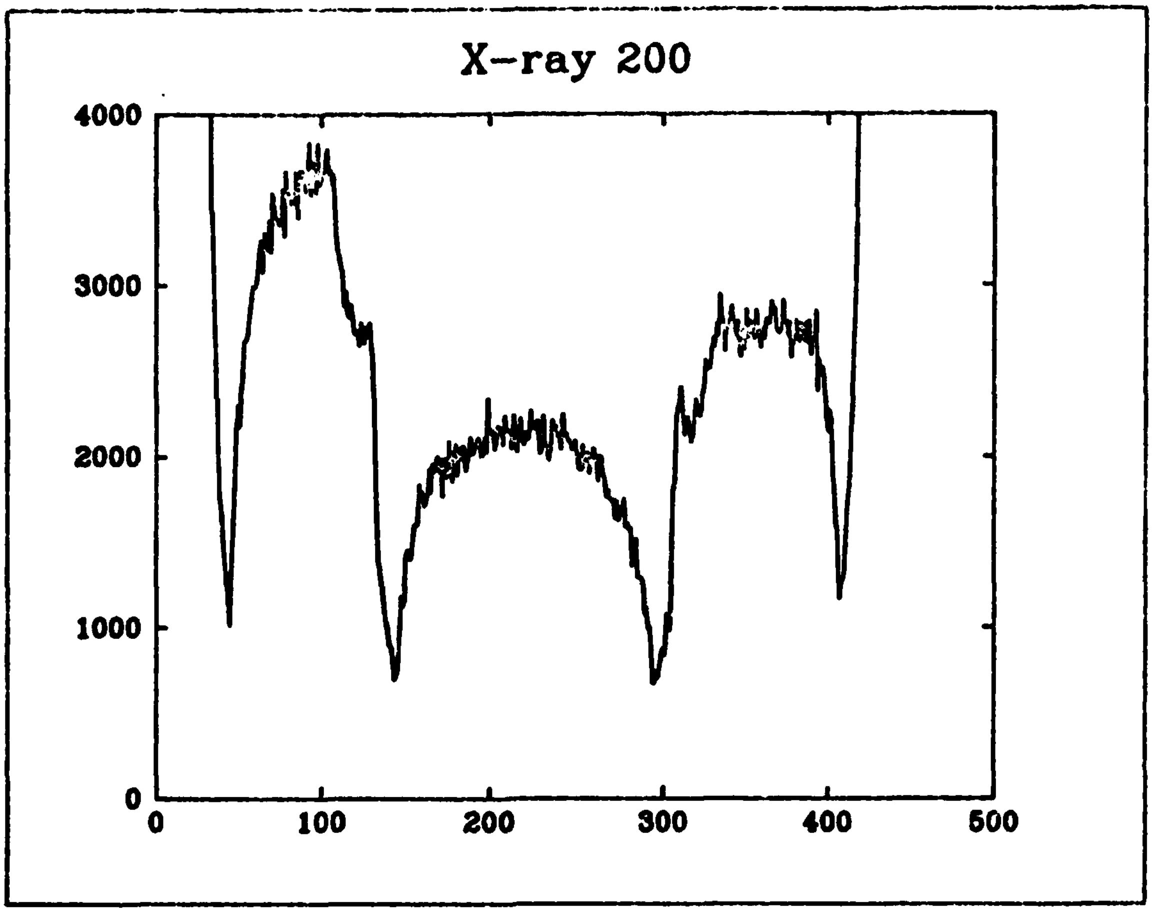
The next graphs below show the noise (i.e. differences) from x-rays 1, 100, and 200, and the two sets of noise added to the phantom. It is this same noise that is added to the phantom in the subsequent sections. (Since the medians appear to give better results, there might be fewer artifacts in noise produced with medians, but it is also useful to have a situation where the noise is not created through the procedure itself. In future experiments the noise will come from x-rays of an explicitly known phantom.) The final graphs show the noisy and smoothed phantoms with the correct phantom overlaid on the latter.

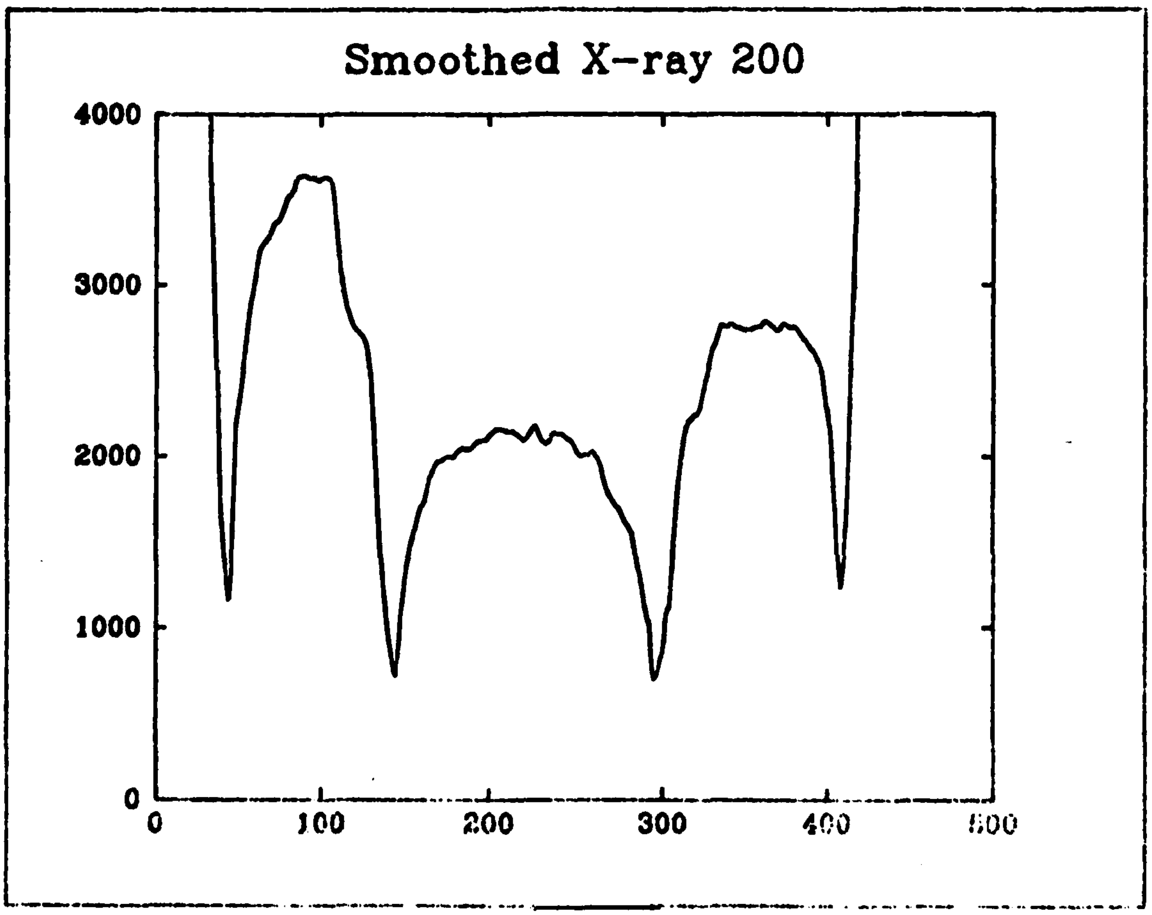

X-ray 1
4000
3000
2000
1000
0
0
100
200
300
400
500

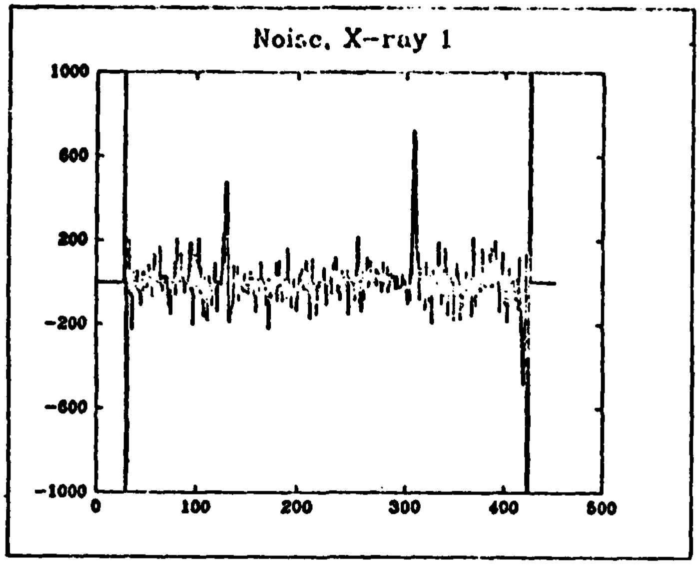

Smoothed X-ray 1
4000
3000
2000
1000
0
0
100
200
300
500

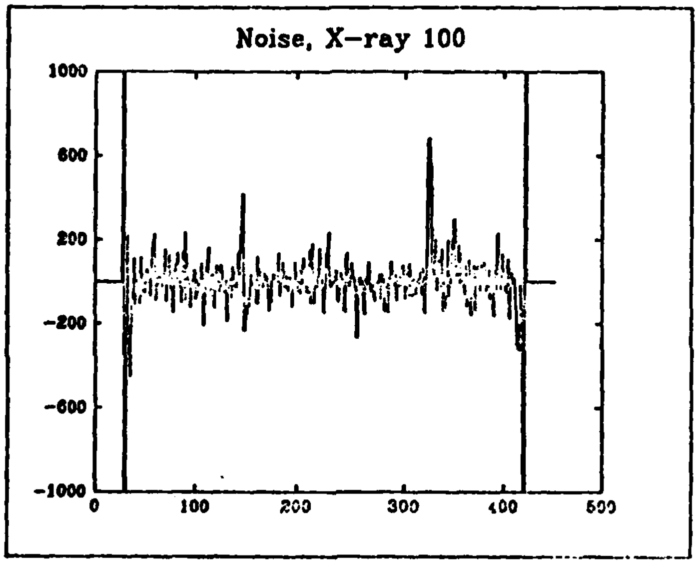

X-ray 100
4000
3000
2000
1000
0
0
100
200
300
400
500

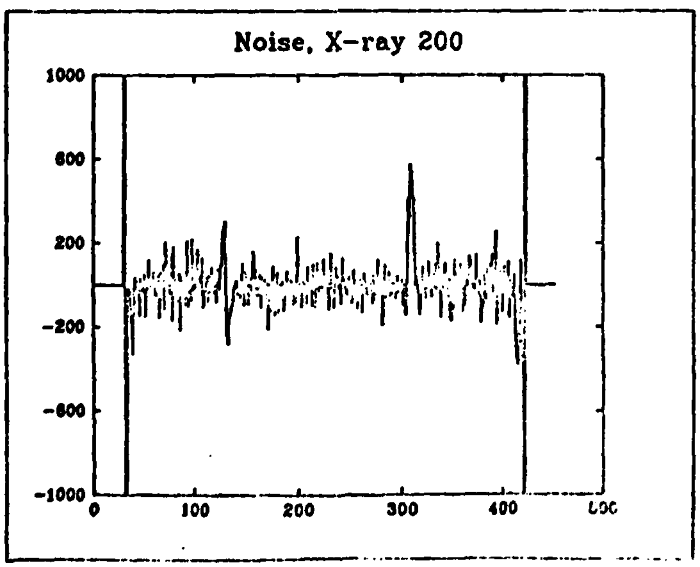

Smoothed X-ray 100
4000
3000
2000
1000
0
0
100
200
300
400
500

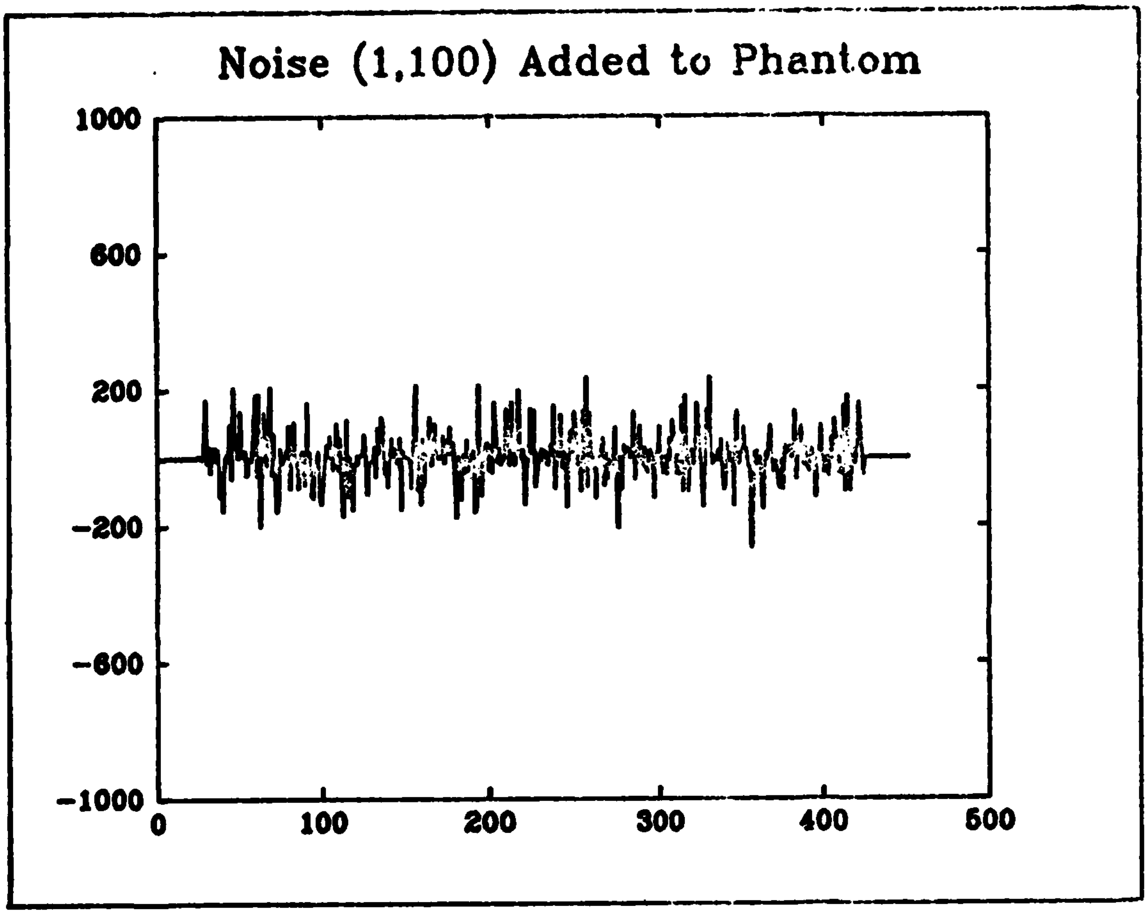

X-ray 200
4000
3000
2000
1000
0
0
100
200
300
400
500


Smoothed X-ray 200
4000
3000
2000
1000
0
0
100
200
300
400
500

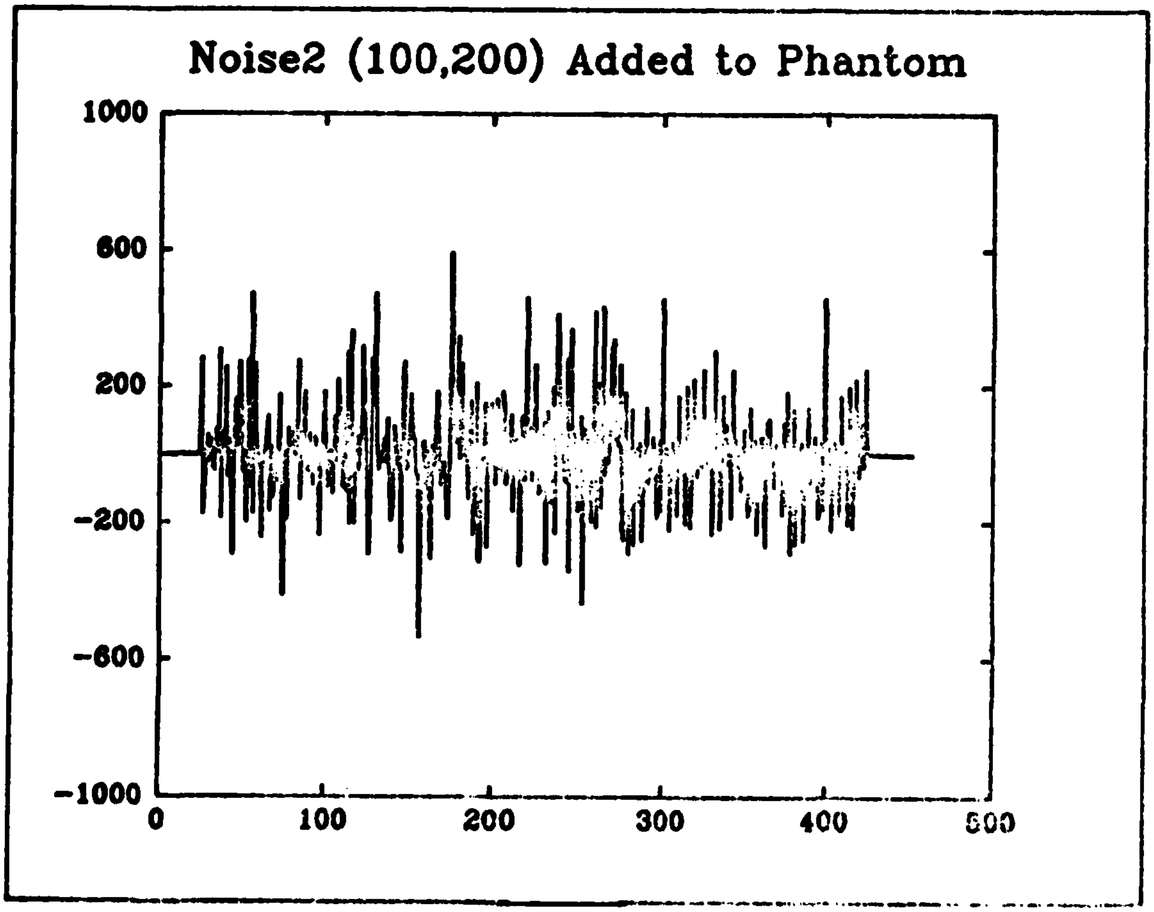

Noise, X-ray 1
1000
600
200
-200
-600
-1000
0
100
200
300
400
500


Noise, X-ray 100
1000
600
200
-200
-600
-1000
0
100
200
300
400
500

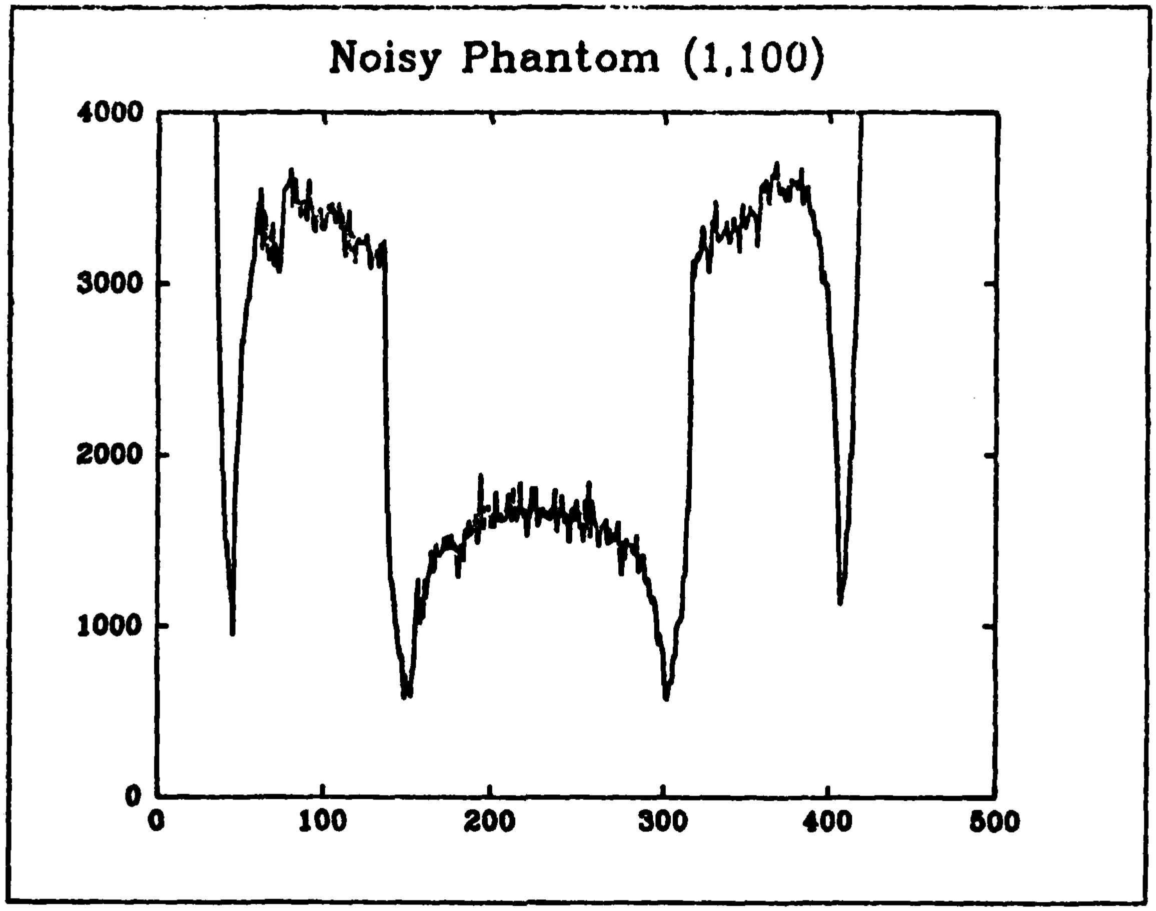

Noise, X-ray 200
1000
600
200
-200
-600
-1000
0
100
200
300
400

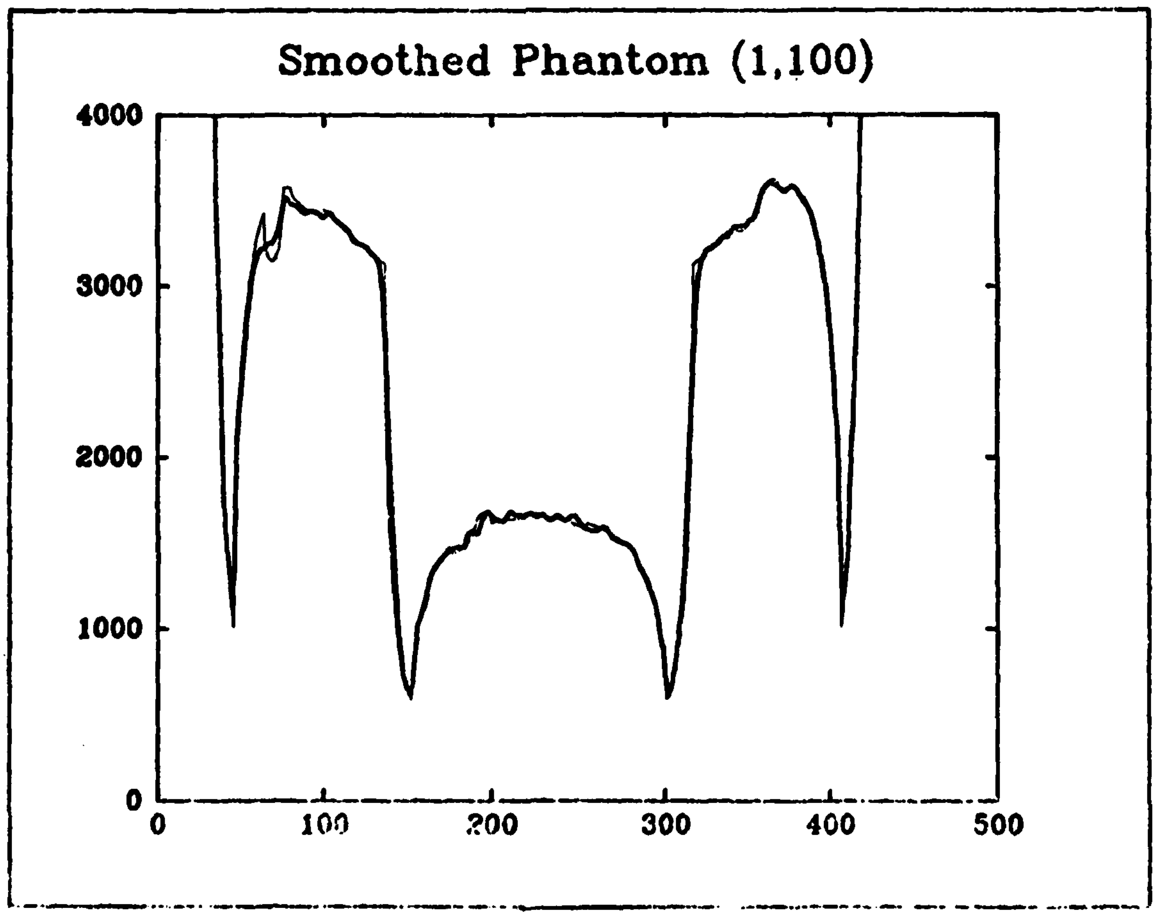


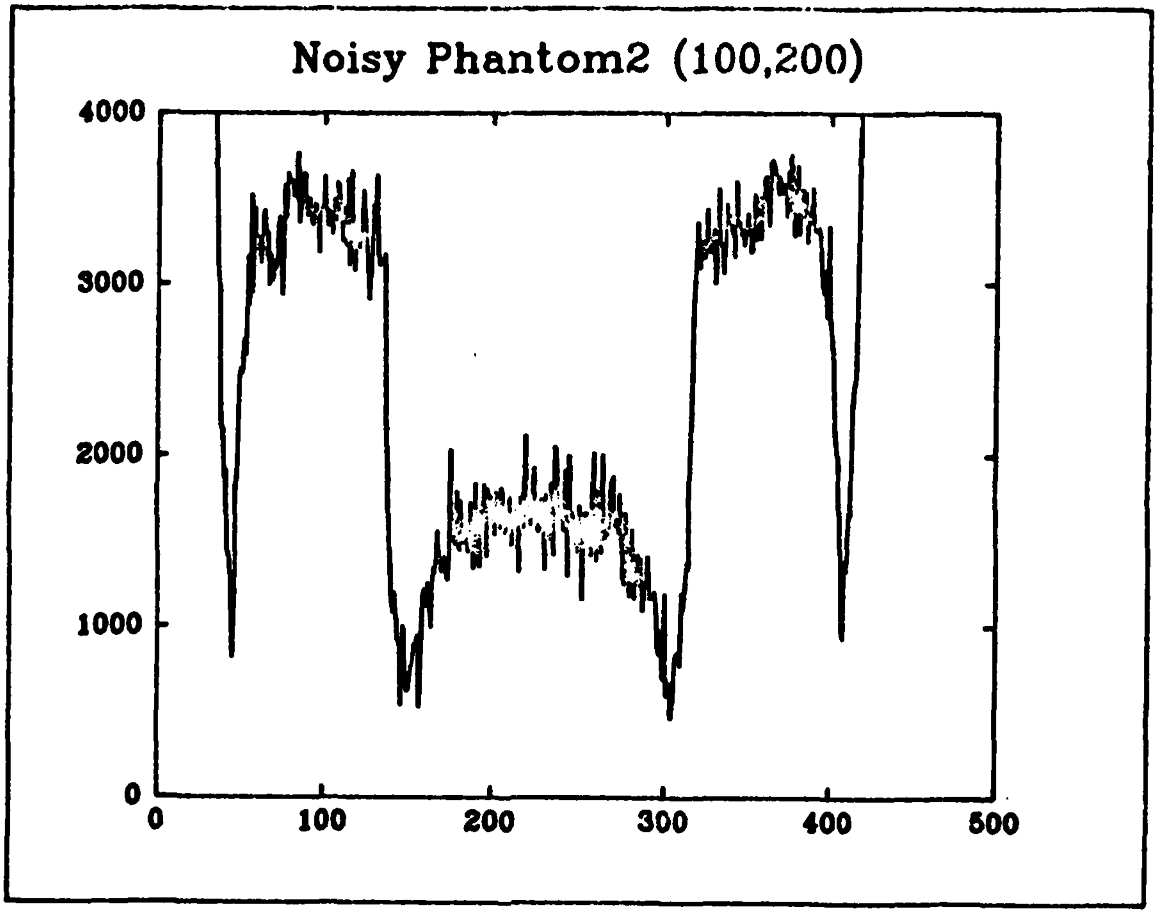
Noise (1,100) Added to Phantom
1000
600
200
-200
-600
-1000
0
100
200
300
400
500

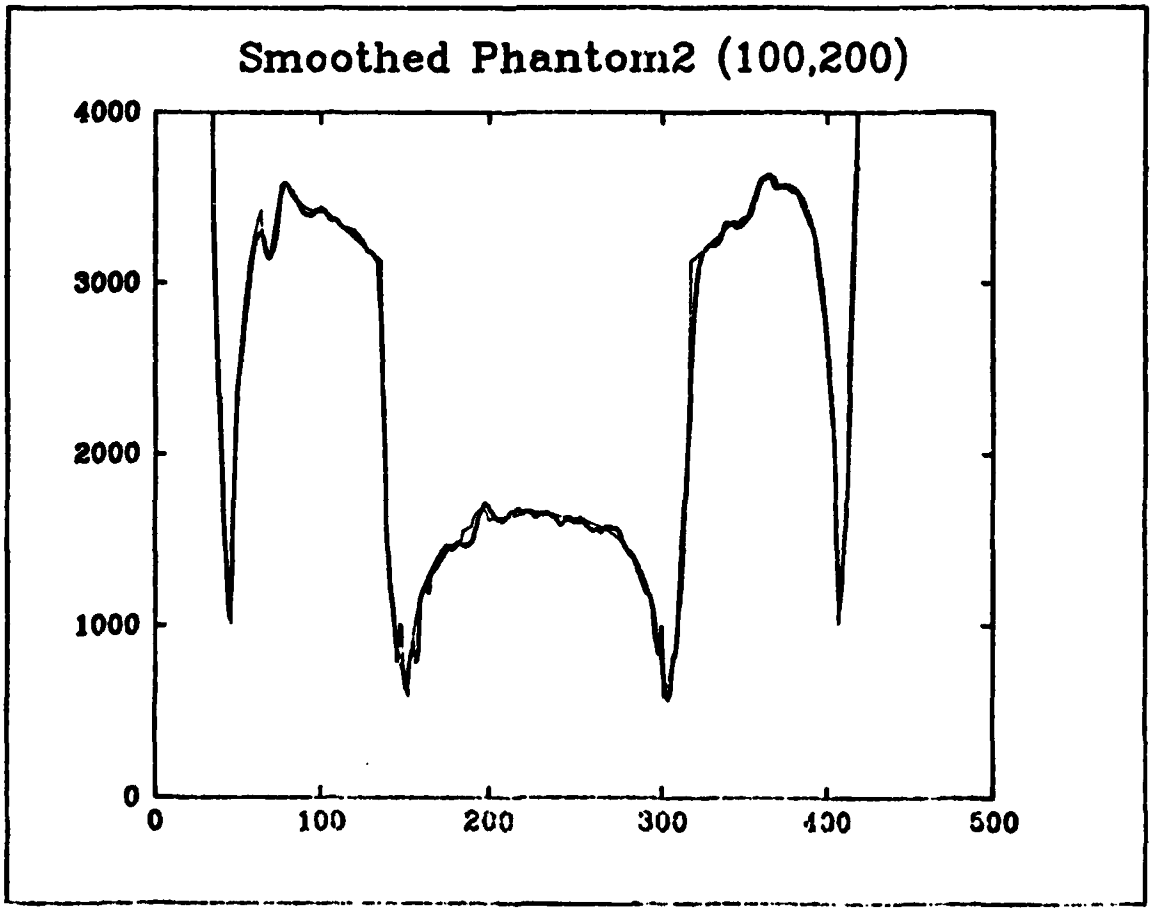


Noise2 (100,200) Added to Phantom
1000
600
200
-200
-600
-1000
0
100
200
300
400
500


Noisy Phantom (1,100)
4000
3000
2000
1000
0
0
100
200
300
400
500


Smoothed Phantom (1,100)
4000
3000
2000
1000
0
0
100
200
300
400
500

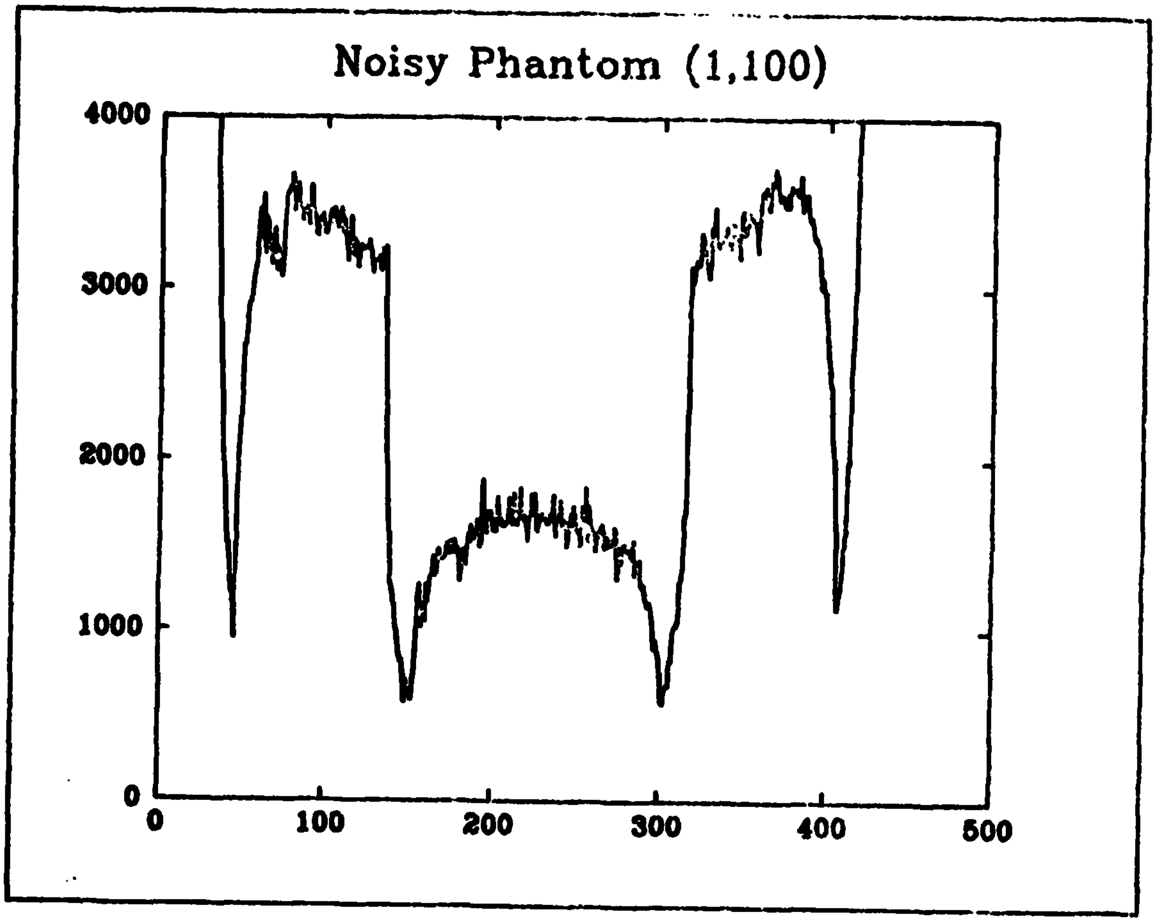

Noisy Phantom2 (100,200)
4000
3000
2000
1000
0
0
100
200
300
400
500

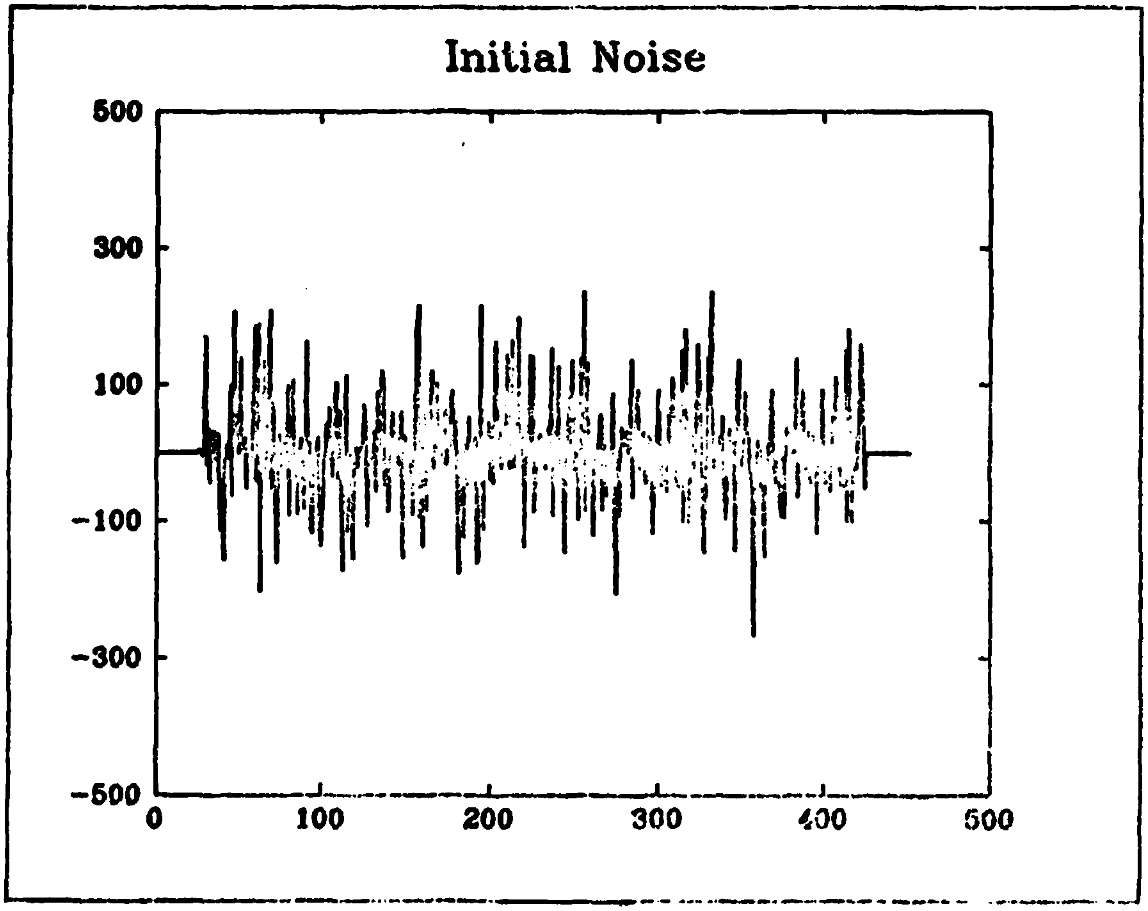

Smoothed Phantom2 (100,200)
4000
3000
2000
1000
0
0
100
200
300
400
500

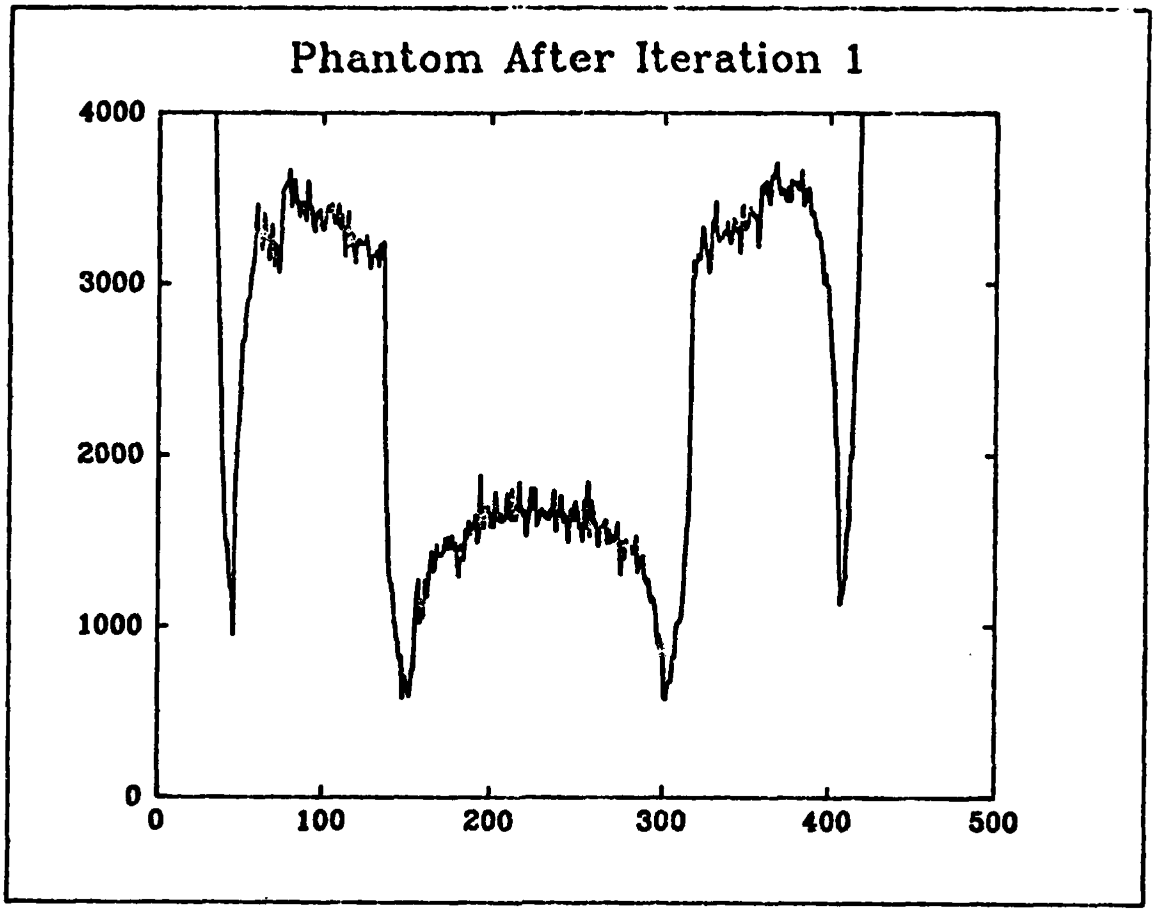
5. Smoothing with averages and medians.

In this section the smoothing is done with both averages and medians: AL_k, AR_k, and A_k are the left, right, and symmetric averages described in section 4, while $D_{k+1}(i)$ is the average of the three middle terms in the nondecreasing rearrangement of D_k over the interval of length $2L_k+1$ with center at i. This means that the points where corrections are to be made are identified using averages, and the corrections are then made using (essentially) medians.

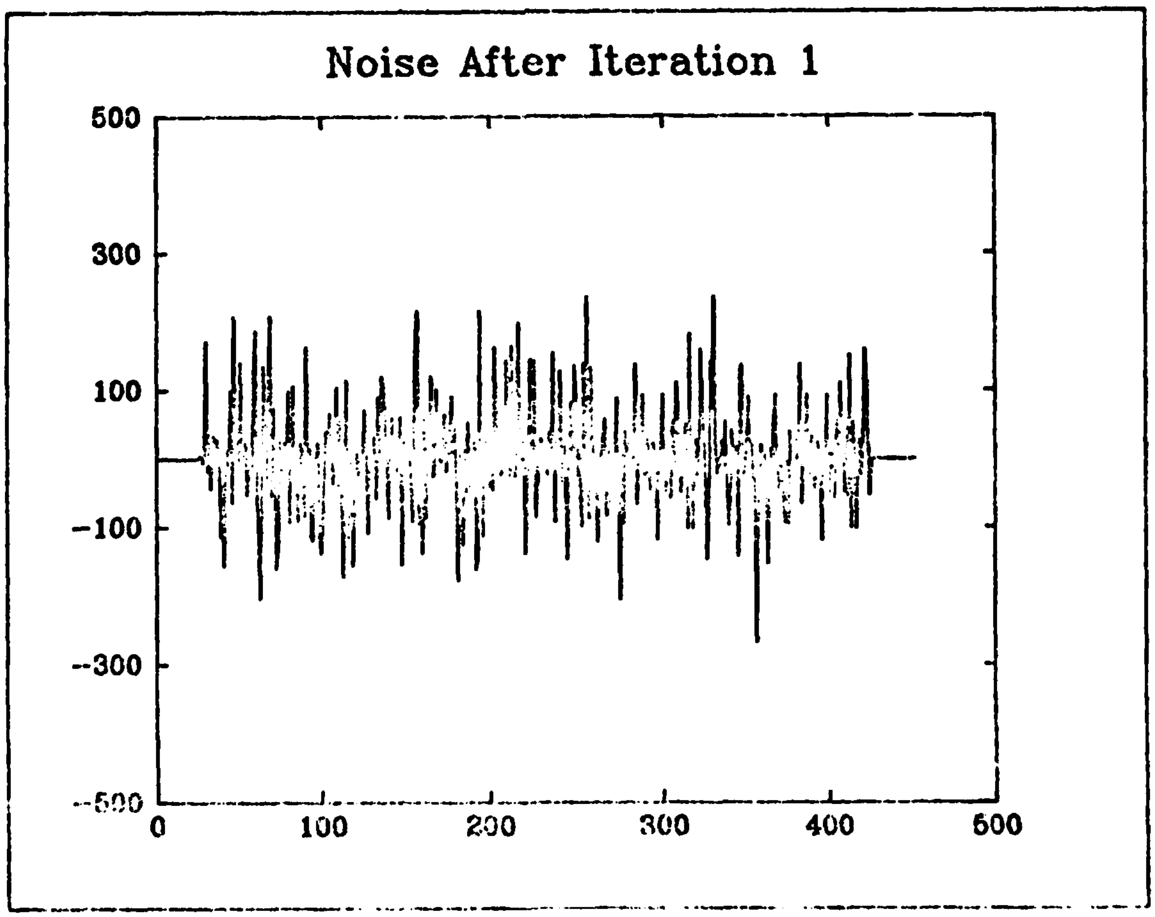
The graphs below show the phantom and the noise at each step of the process. The noise is now the difference between the current phantom and the true phantom. The points where corrections are made (very few in the early steps) can be identified by holding successive graphs together against a light. It will be observed that these are not the points where the current true noise is largest, but that they are situated near the steel peaks. In other words, the magnitude of the noise as measured by the process is dependent on the setting - as would be expected. This dependence on the setting is reduced when the magnitude is measured using medians, as it is in the next section.


Noisy Phantom (1,100)
4000
3000
2000
1000
0
0
100
200
300
400
500

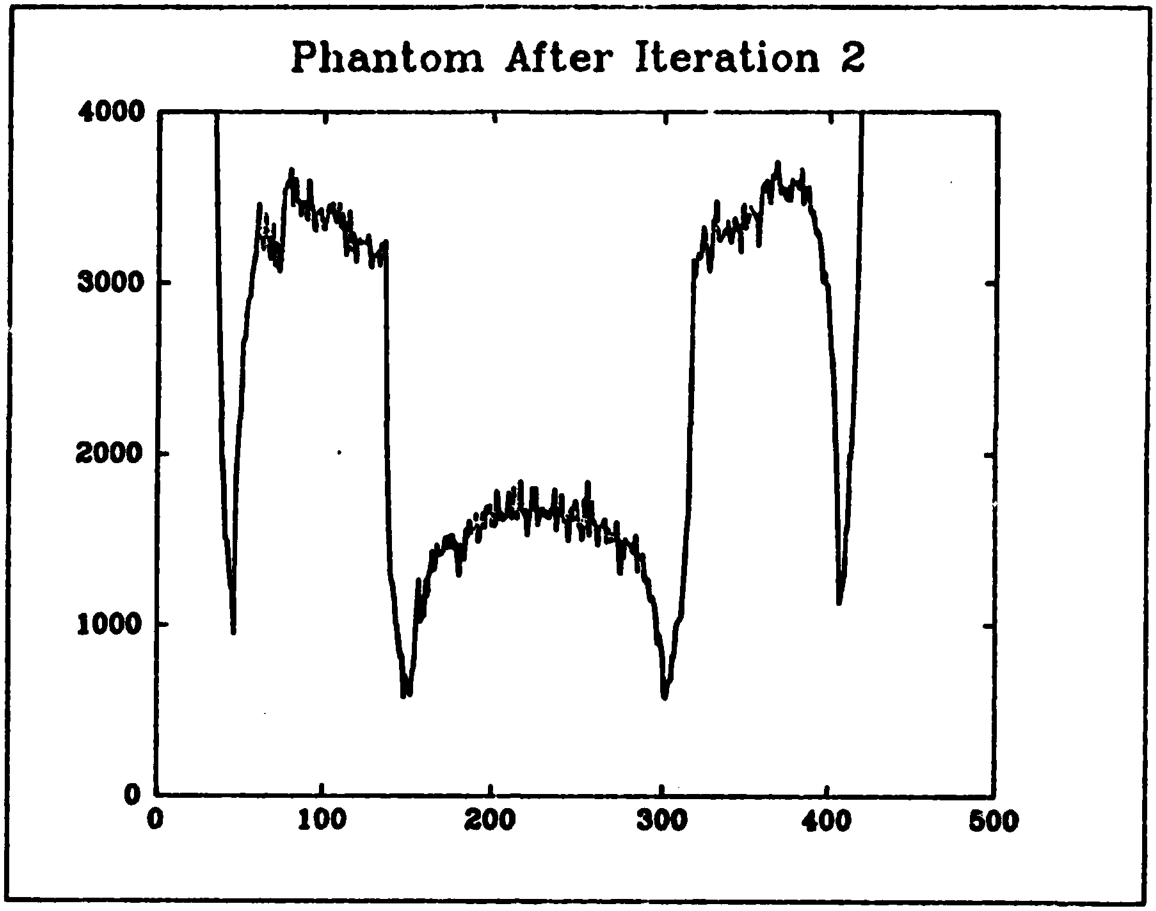

Initial Noise
500
300
100
-100
-300
-500
0
100
200
300
400
500

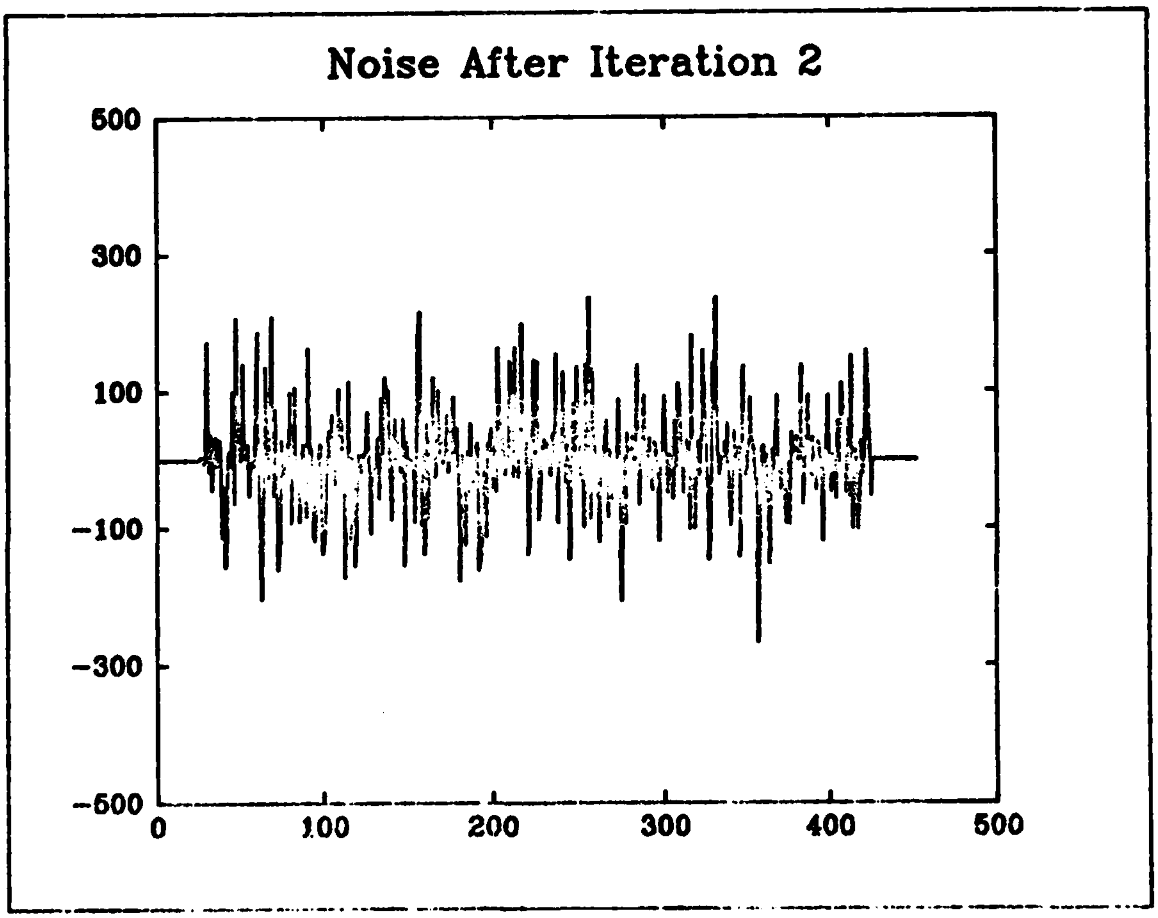


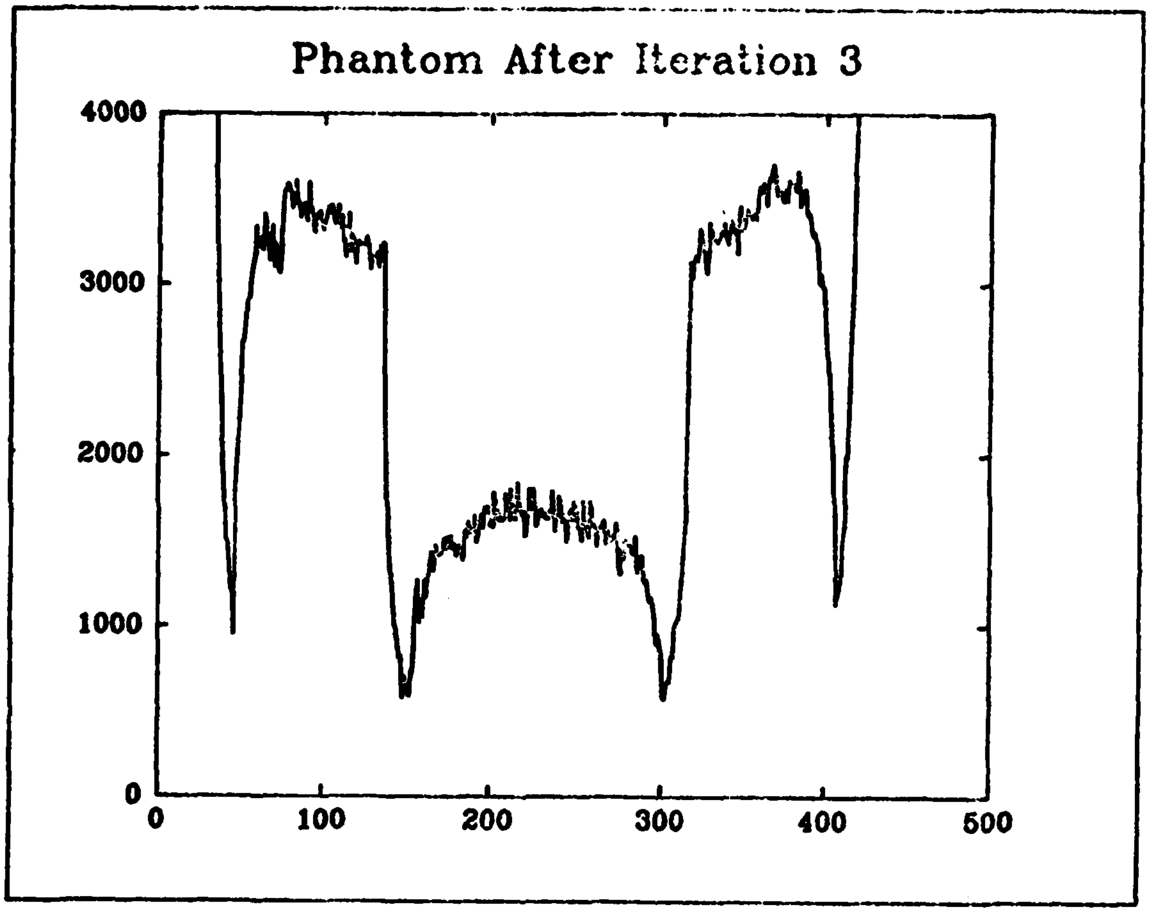
Phantom After Iteration 1
4000
3000
2000
1000
0
0
100
200
300
400
500

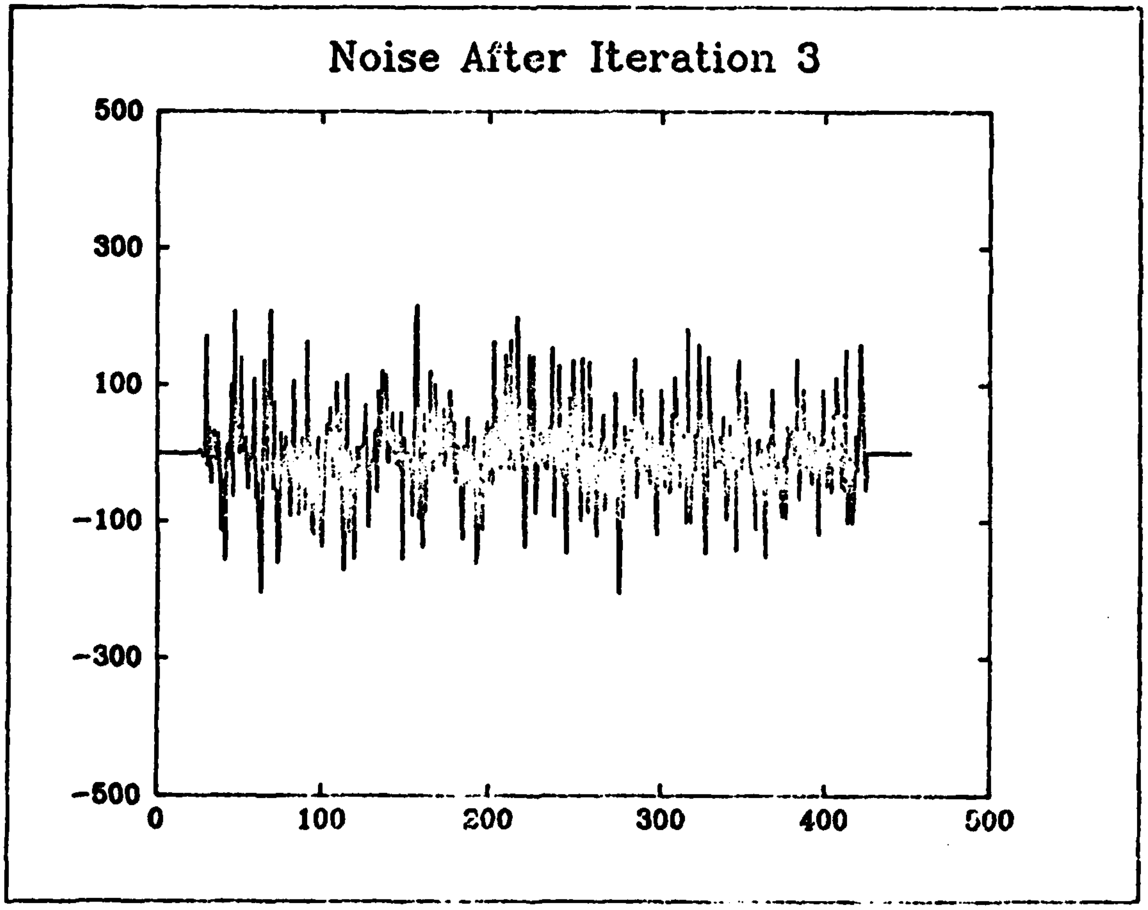


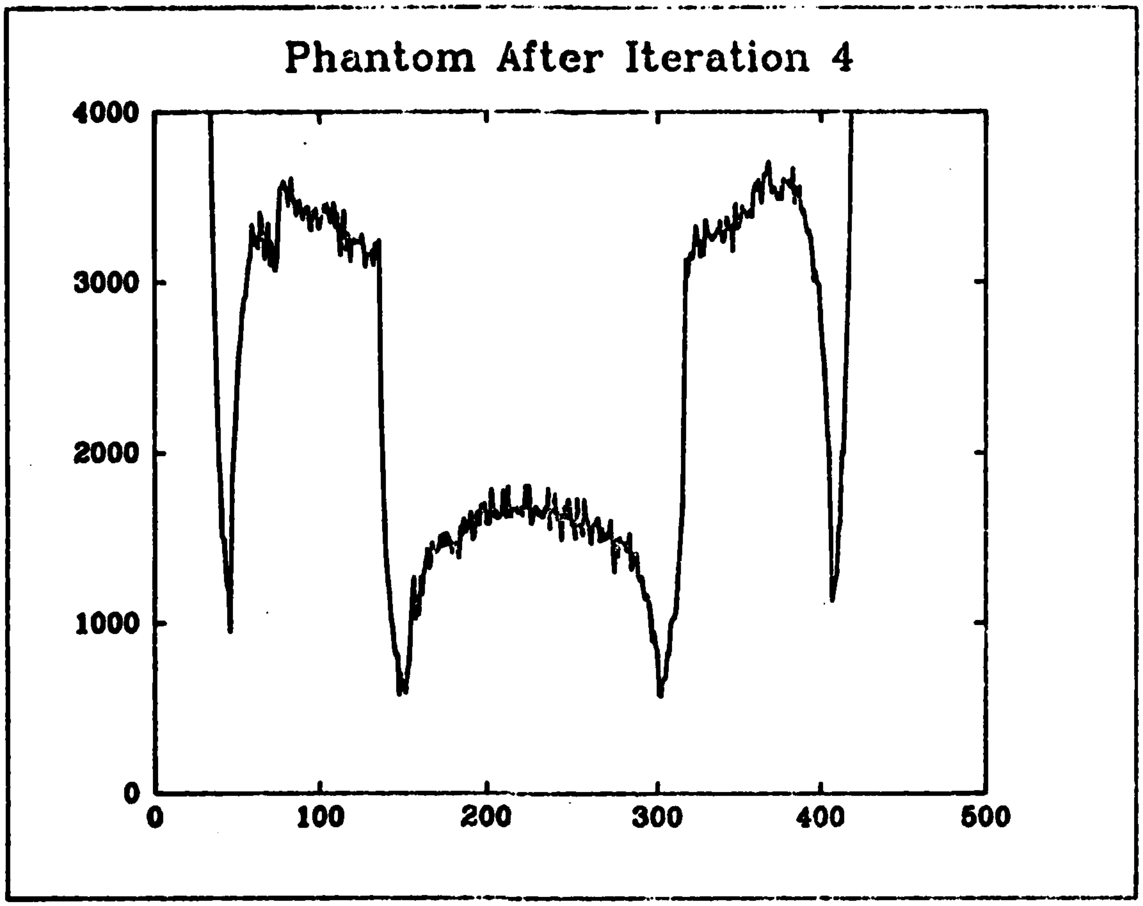
Noise After Iteration 1
500
300
100
-100
-300
-500
0
100
200
300
400
500

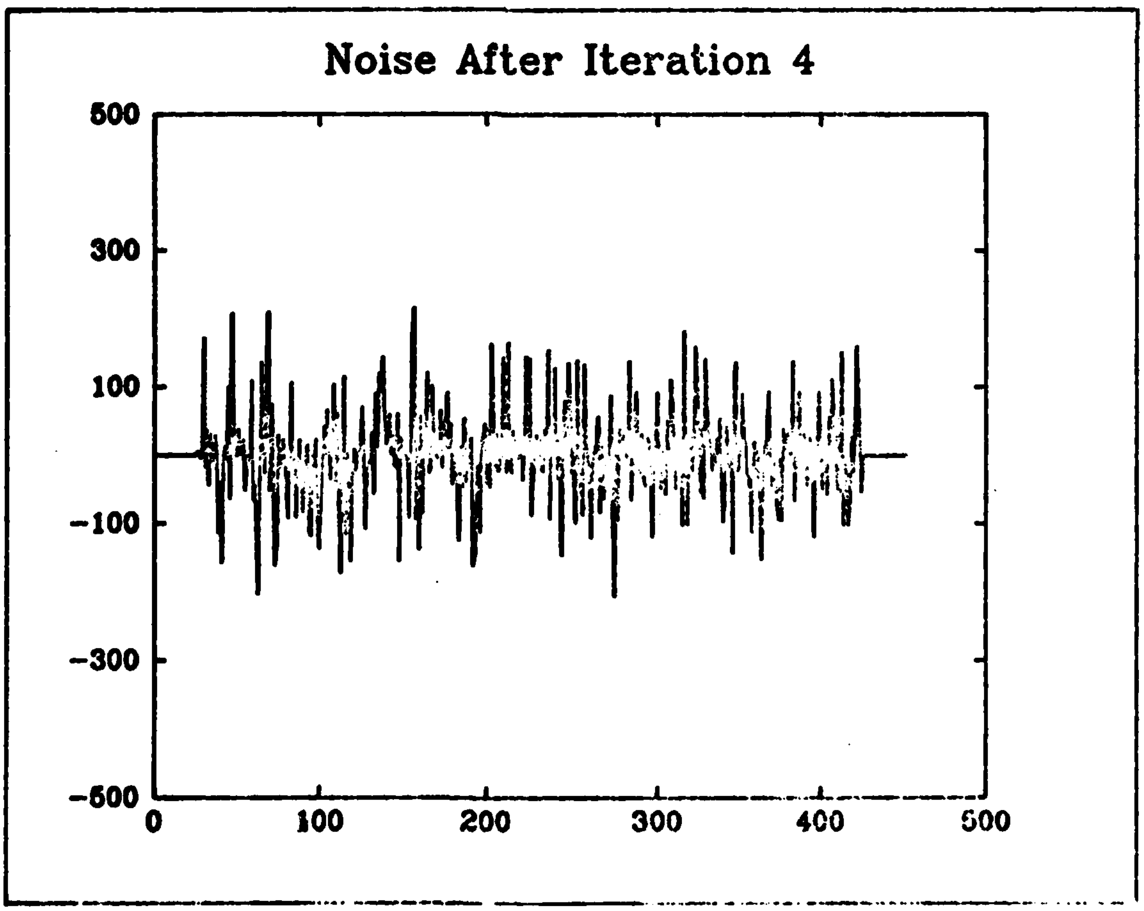

Phantom After Iteration 2
4000
3000
2000
1000
0
0
100
200
300
400
500

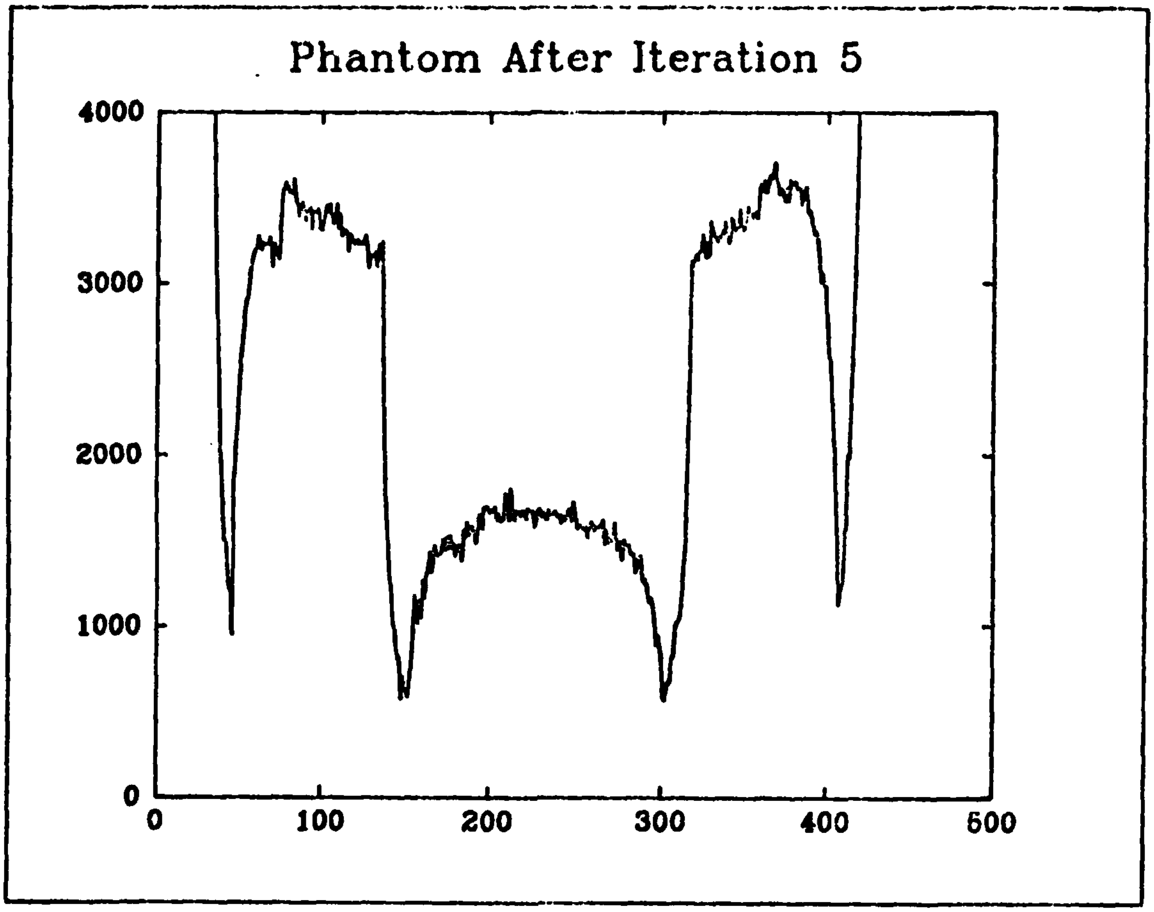

Noise After Iteration 2
500
300
100
−100
−300
−500
0
100
200
300
400
500

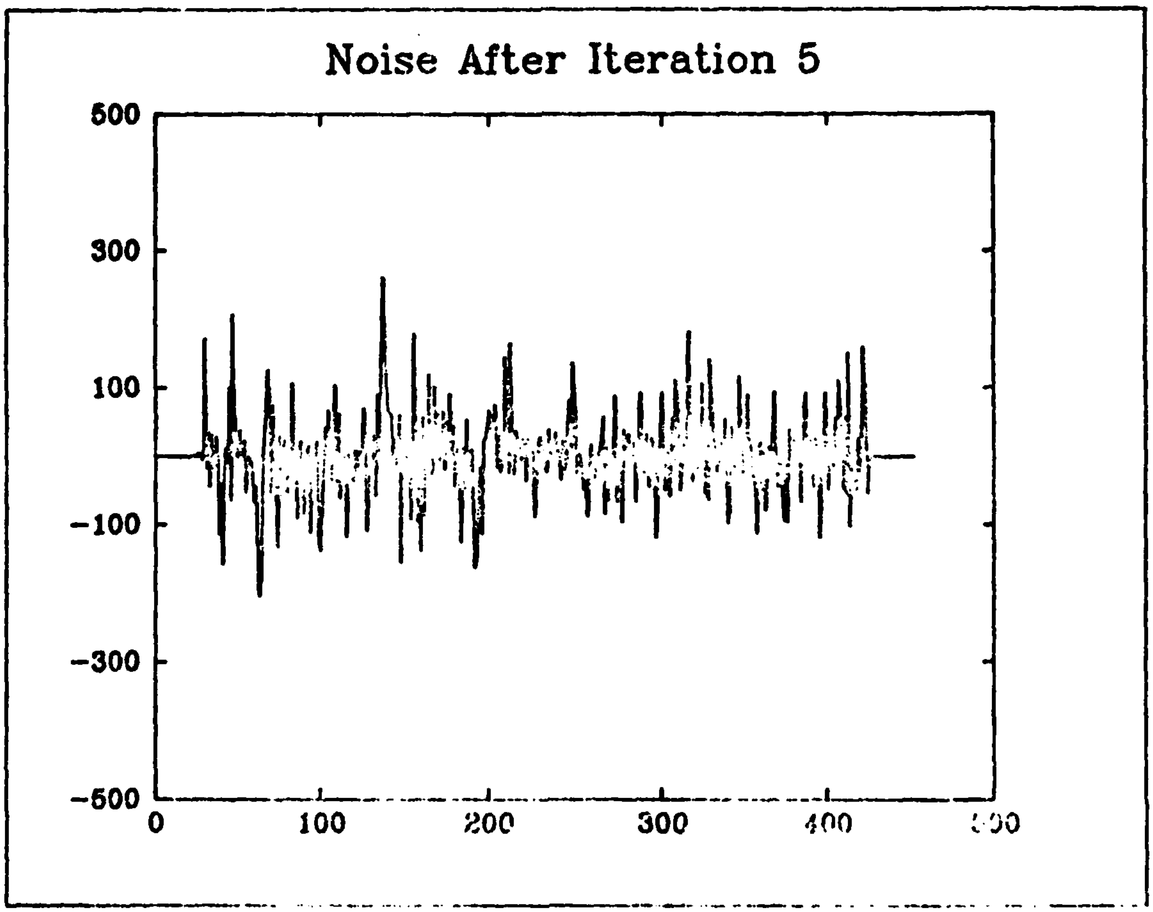

Phantom After Iteration 3
4000
3000
2000
1000
0
0
100
200
300
400
500

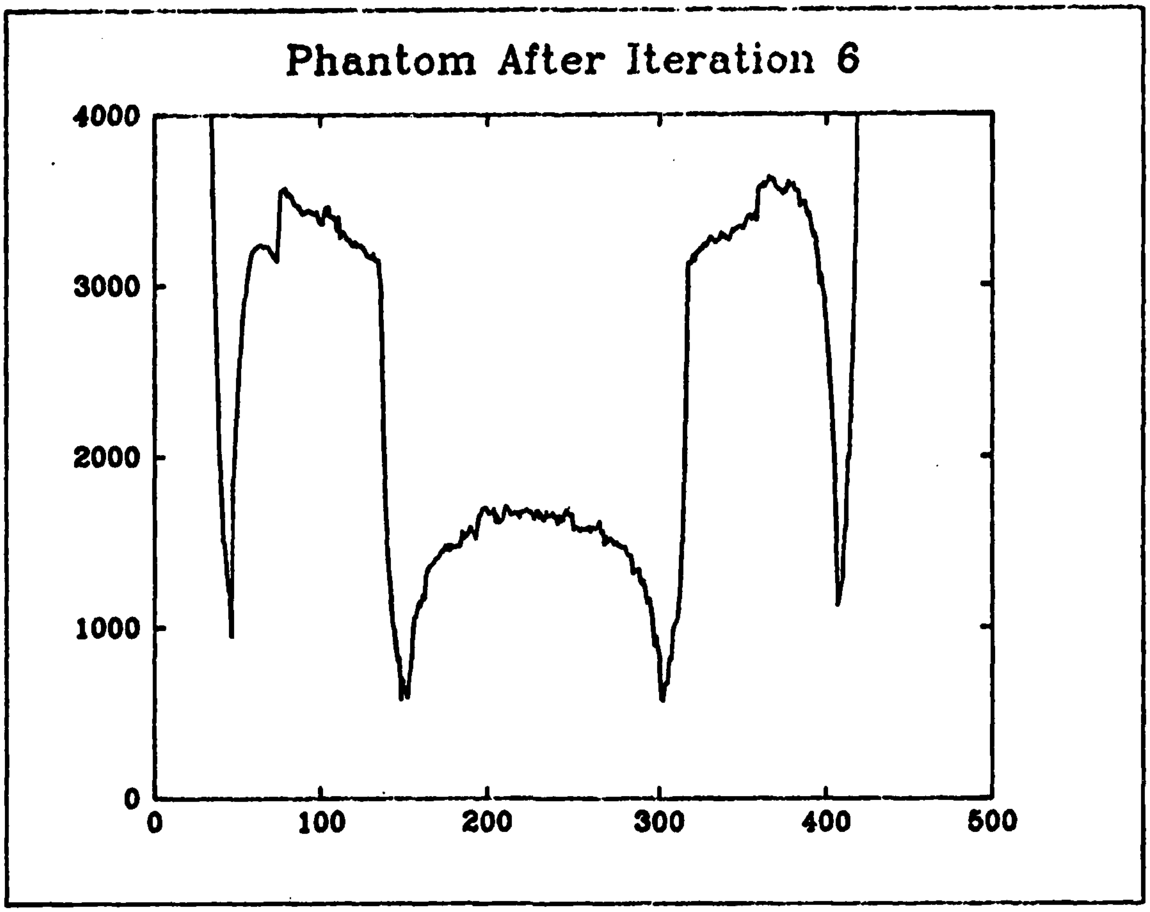

Noise After Iteration 3
500
300
100
−100
−300
−500
0
100
200
300
400
500

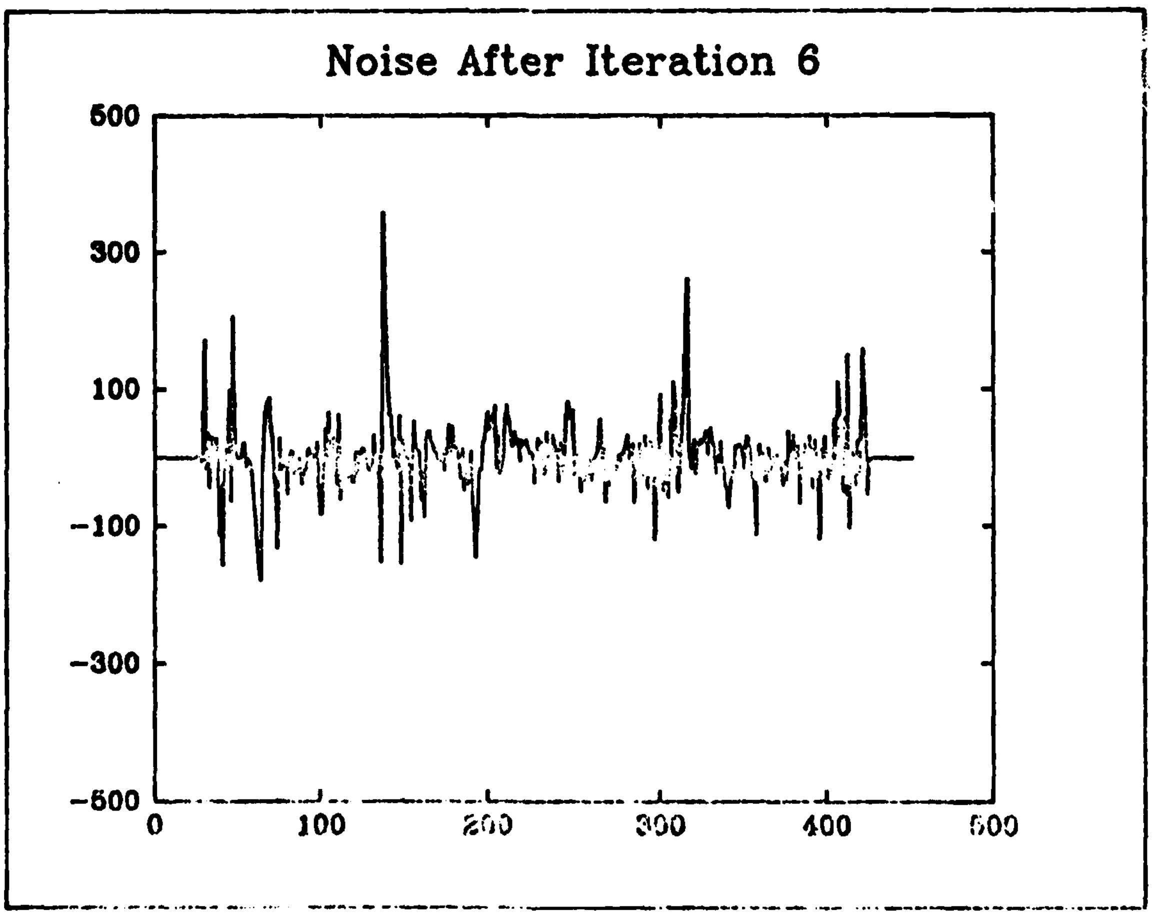

Phantom After Iteration 4
4000
3000
2000
1000
0
0
100
200
300
400
500

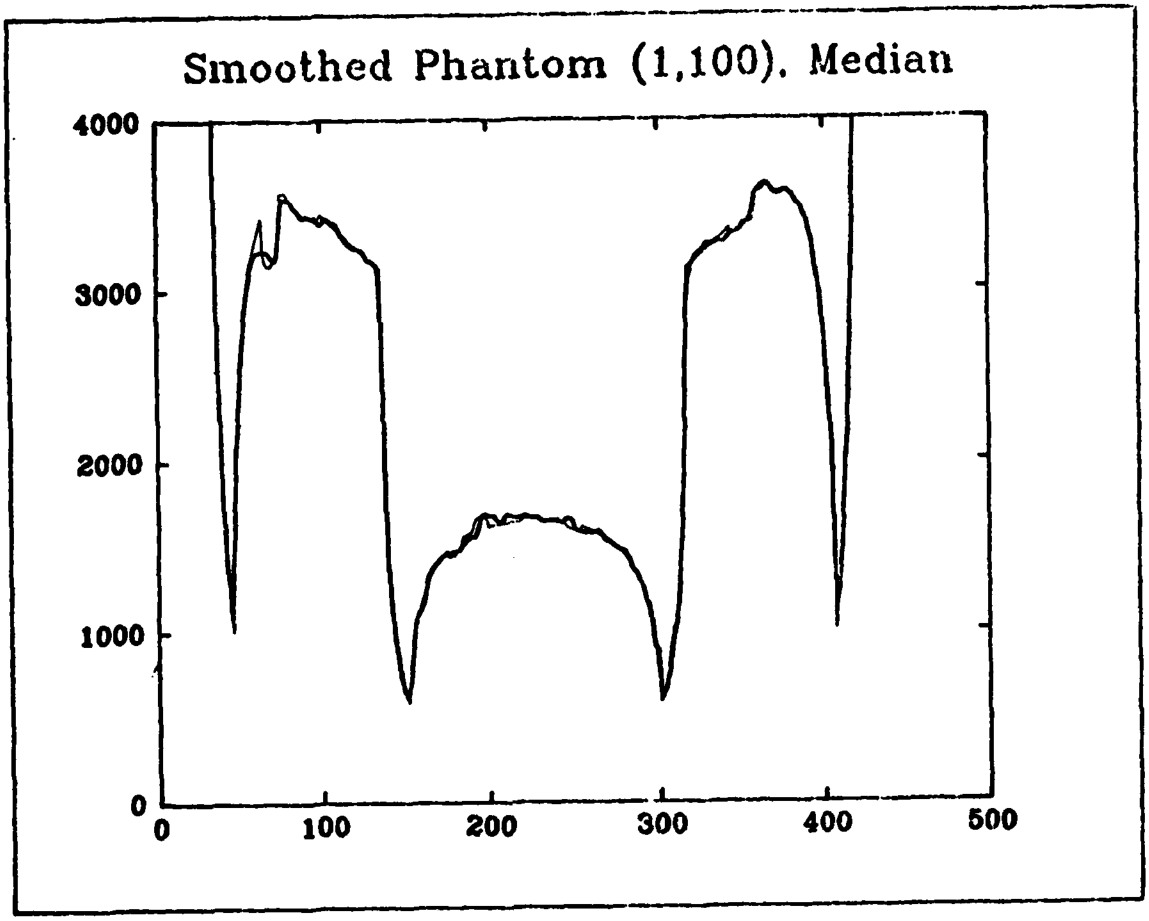

Noise After Iteration 4
500
300
100
−100
−300
−500
0
100
200
300
400
500

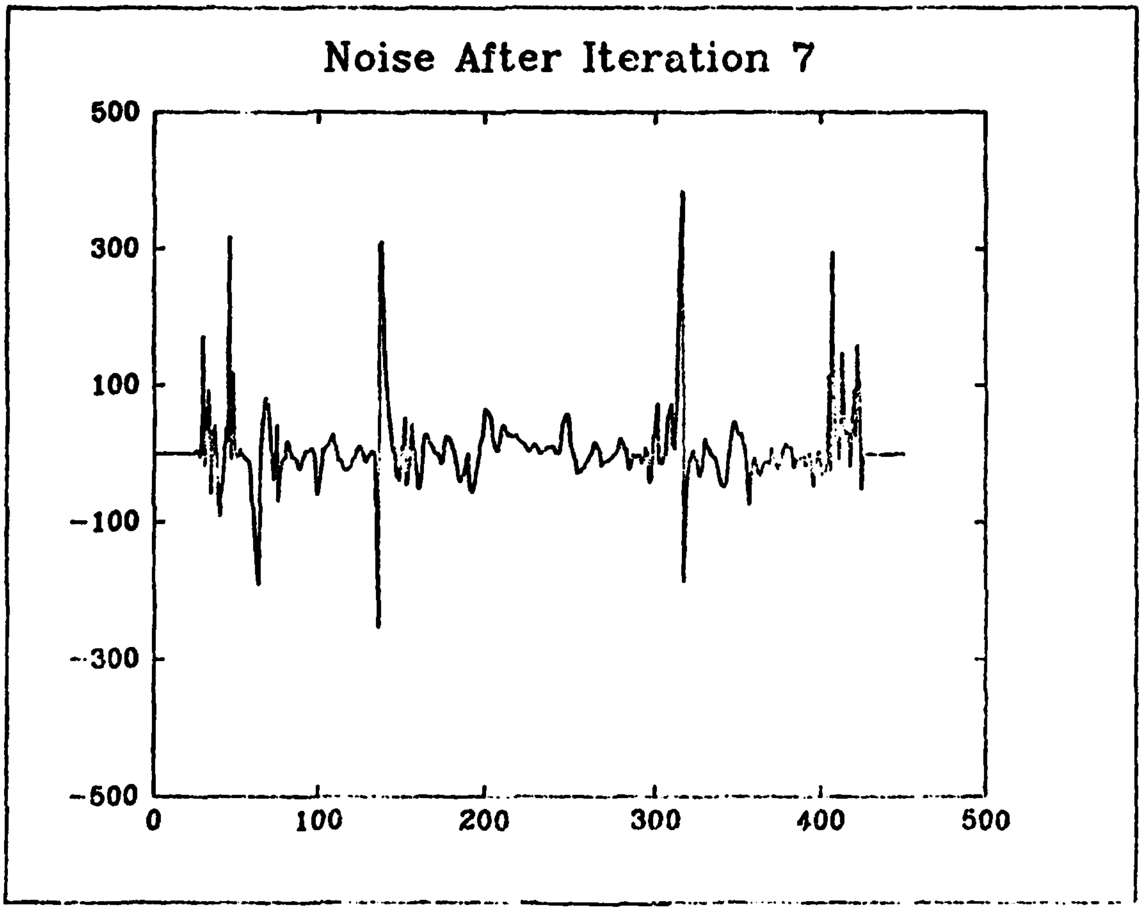

Phantom After Iteration 5
4000
3000
2000
1000
0
0
100
200
300
400
500


Noise After Iteration 5
500
300
100
−100
−300
−500
0
100
200
300
400


Phantom After Iteration 6
4000
3000
2000
1000
0
0
100
200
300
400
500


Noise After Iteration 6
500
300
100
-100
-300
-500
0
100
200
300
400
500

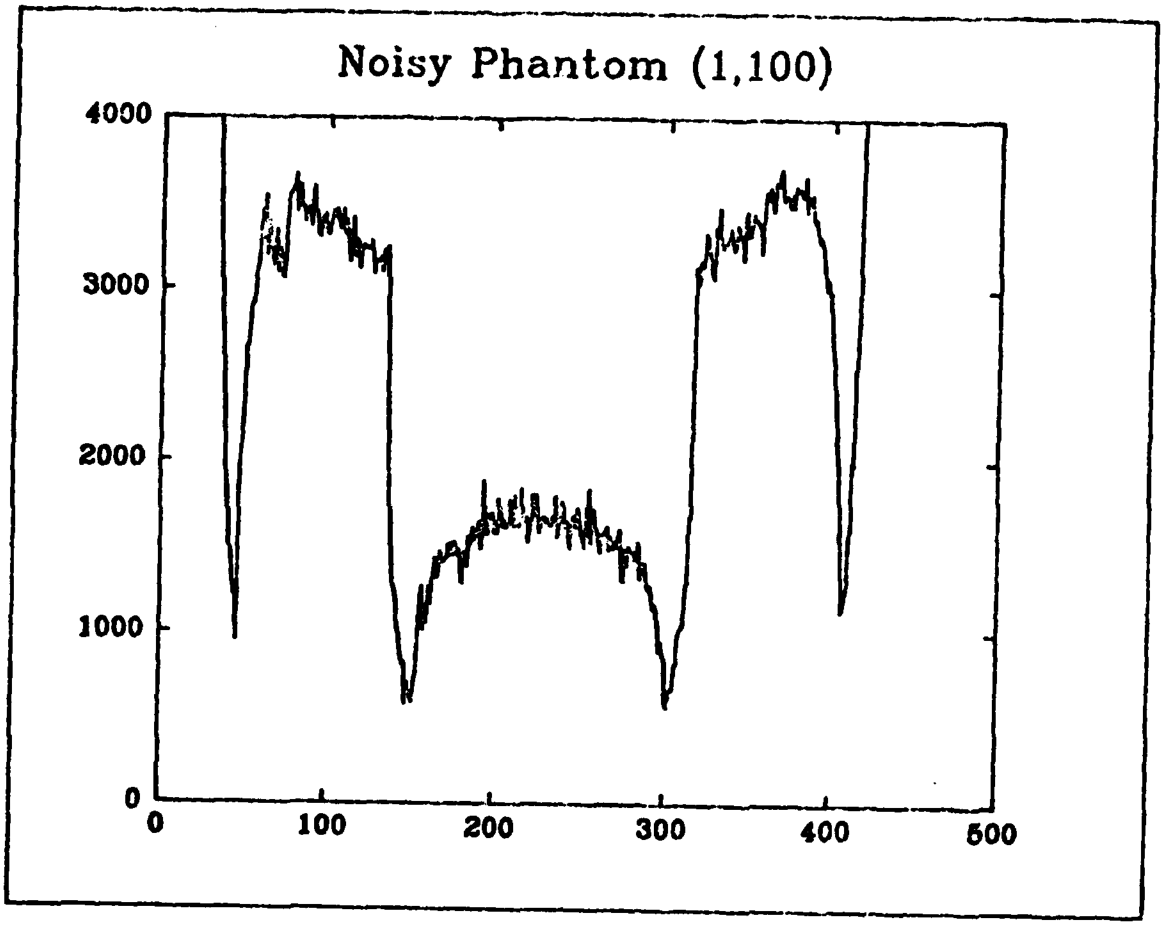

Smoothed Phantom (1,100). Median
4000
3000
2000
1000
0
0
100
200
300
400
500

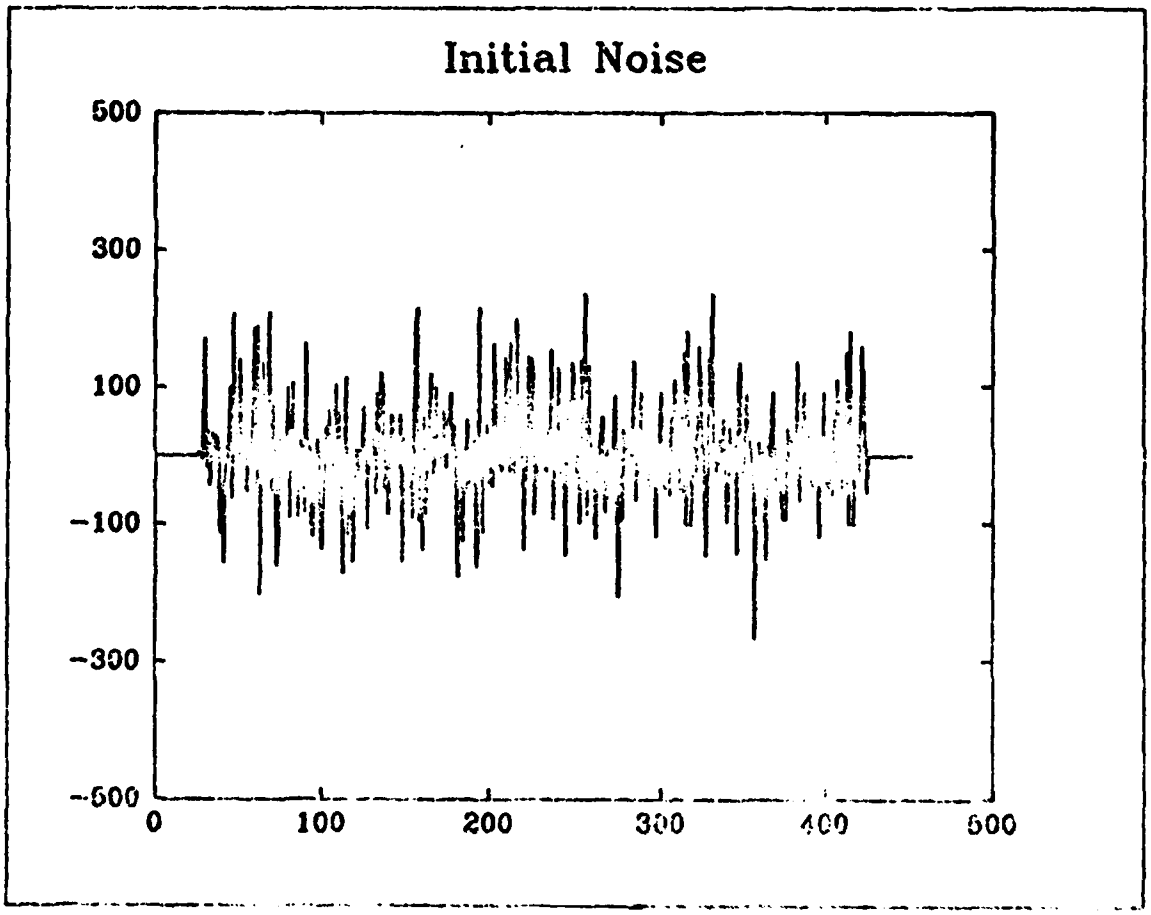

Noise After Iteration 7
500
300
100
−100
−300
−500
0
100
200
300
400
500

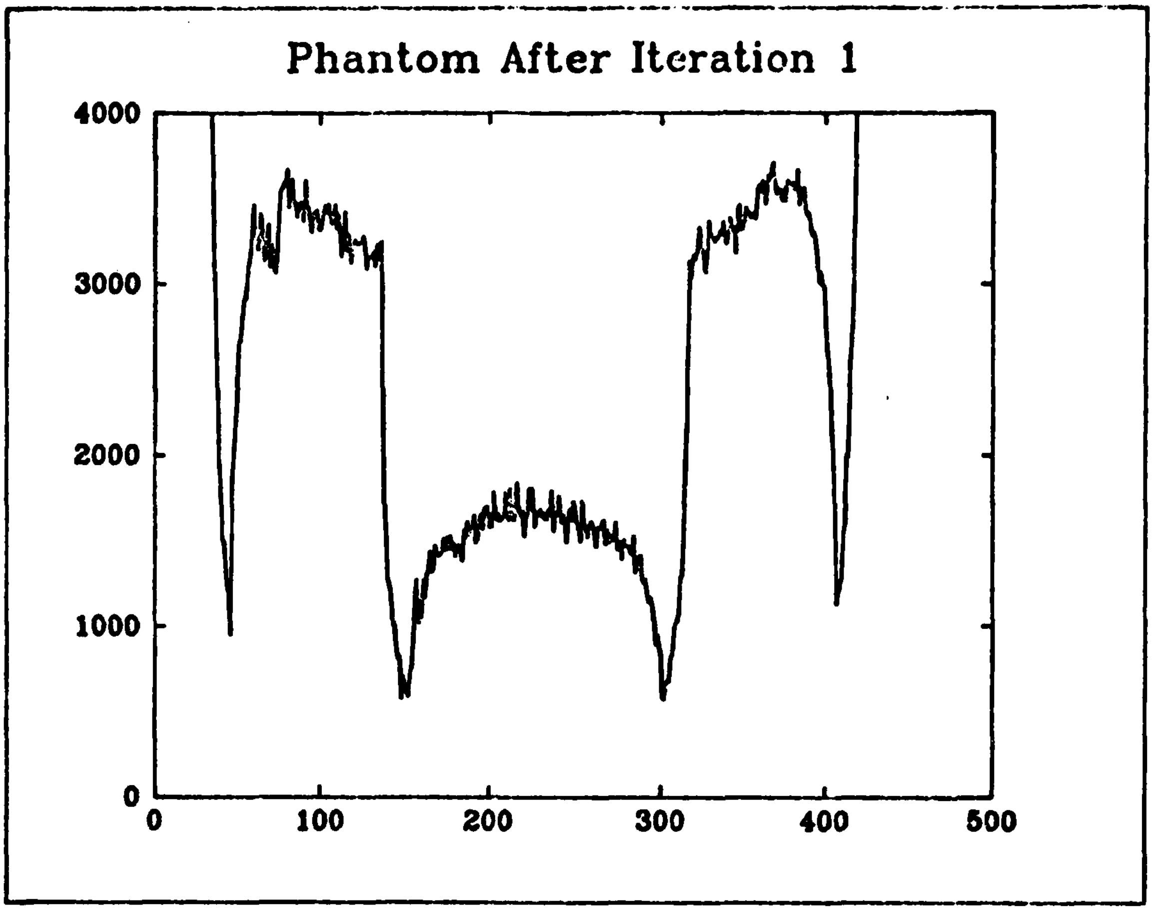
6. Smoothing with medians.

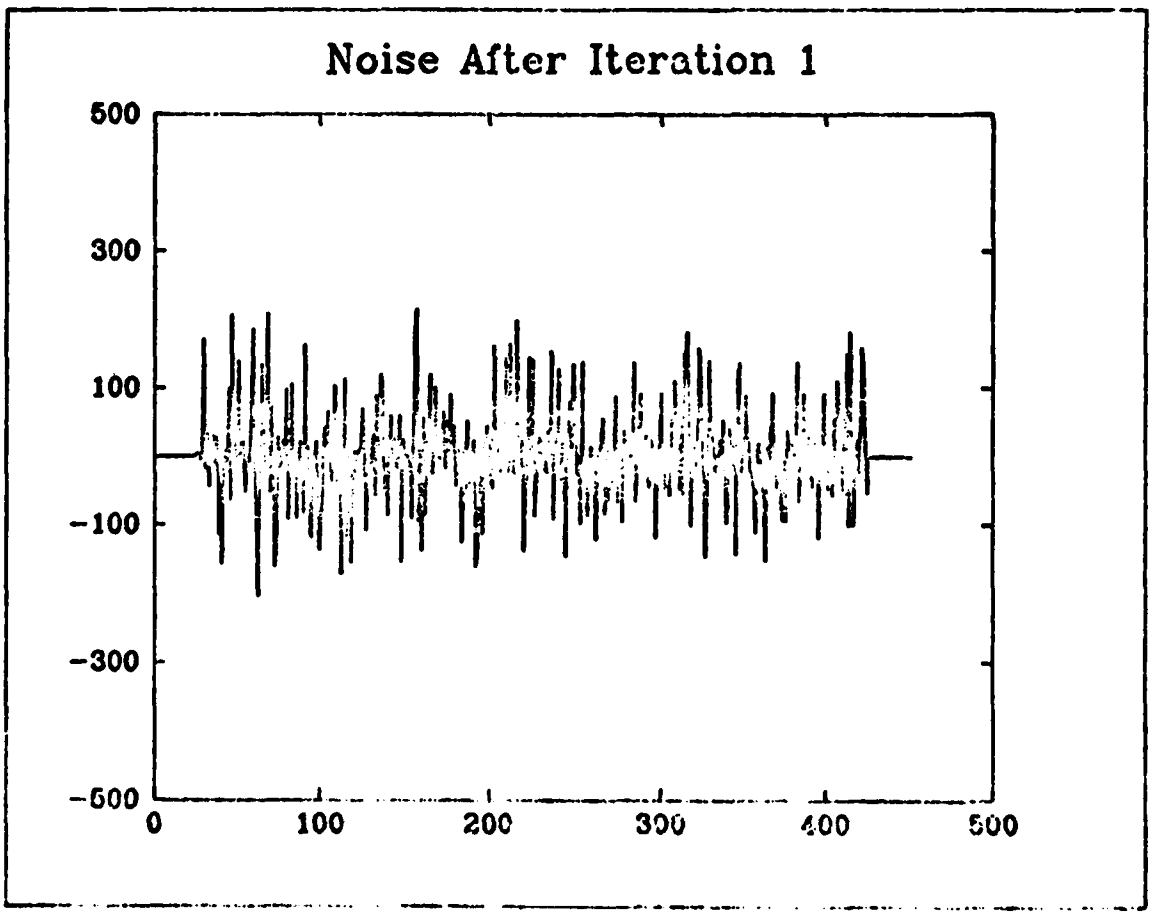
In this section everything is done with medians: $AL_k(i)$ is the average of the two middle terms in the nondecreasing rearrangement of D_k over the interval of length L_k with right endpoint at i, $AR_k(i)$ is the corresponding right hand median, and $A_k(i)$ and $D_{k+1}(i)$ are the symmetric median described in the last section.

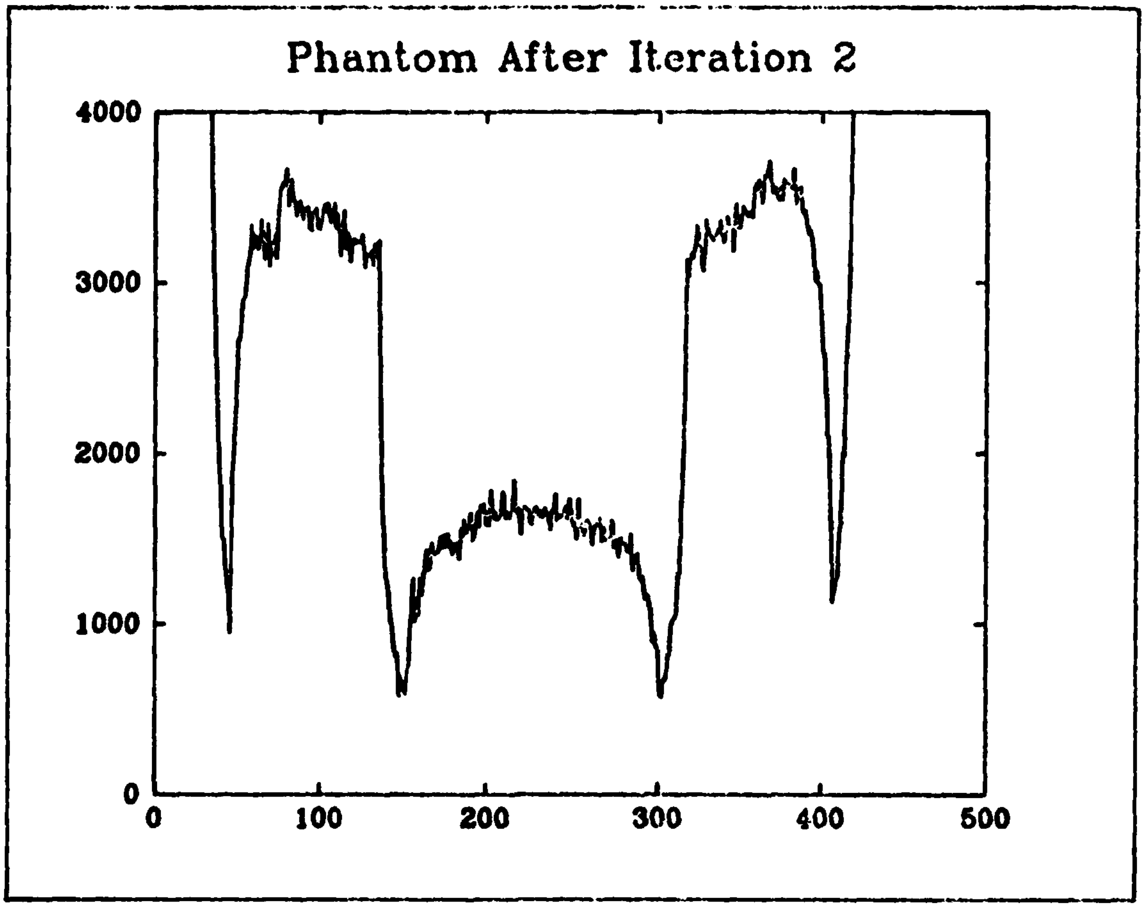
As in the last section, the graphs show the phantom and the noise at each step of the process. It will be observed that the points where corrections are made are now in better accord with the points of large noise.

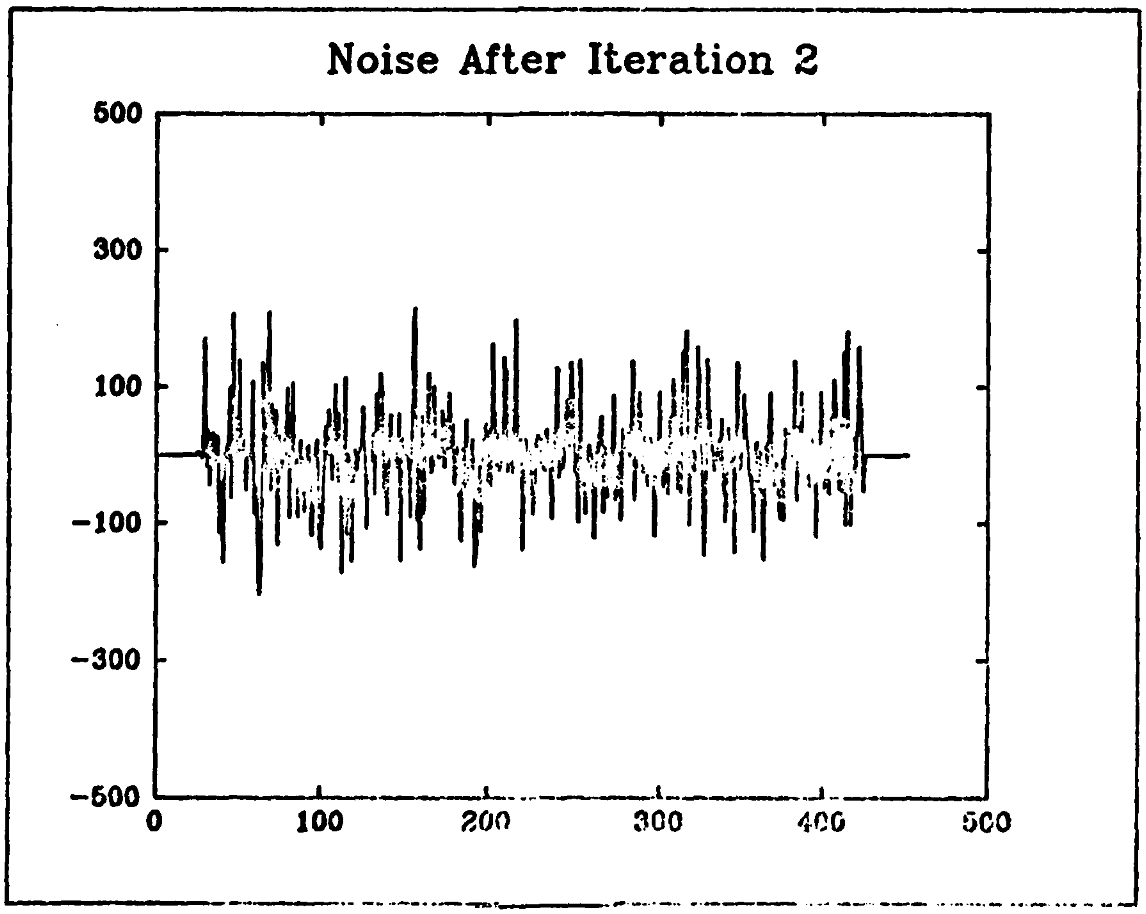

Noisy Phantom (1,100)
4000
3000
2000
1000
0
0
100
200
300
400
500

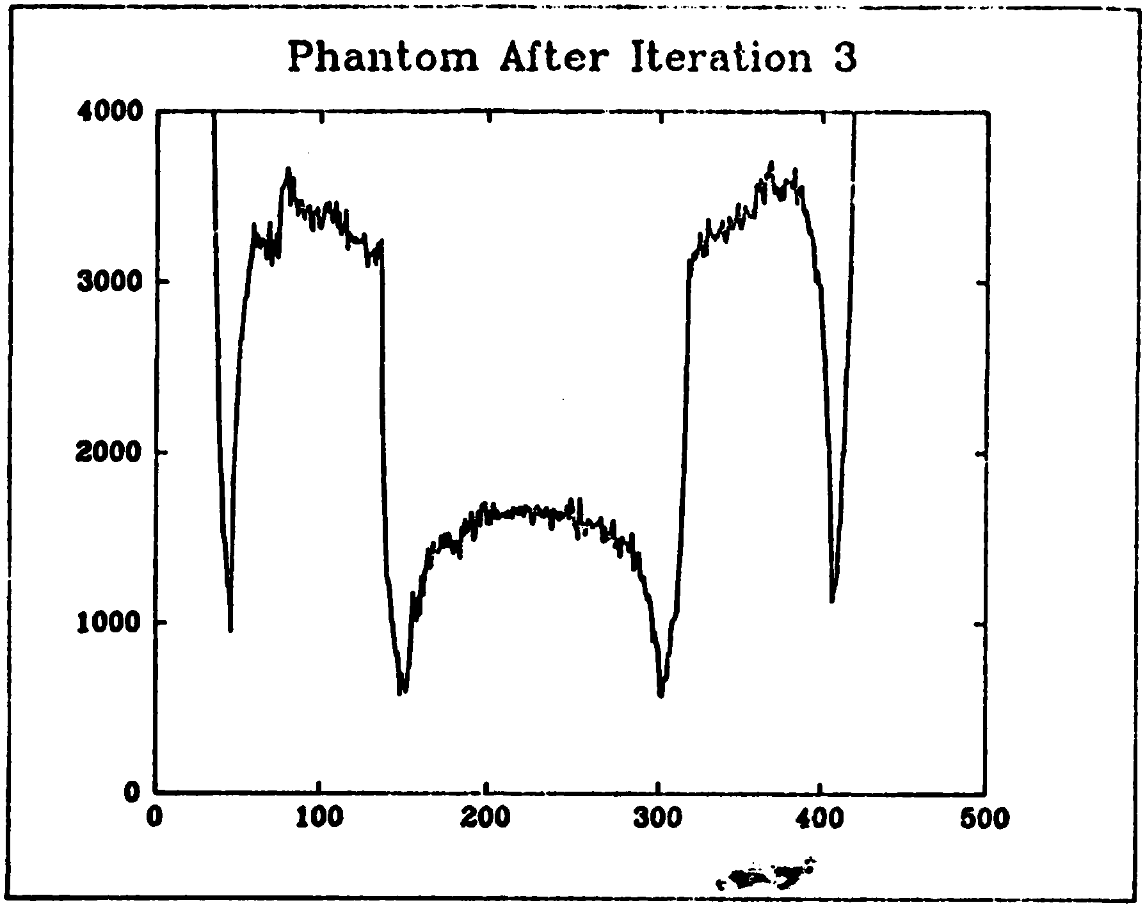

Initial Noise
500
300
100
−100
−300
−500
0
100
200
300
400
500

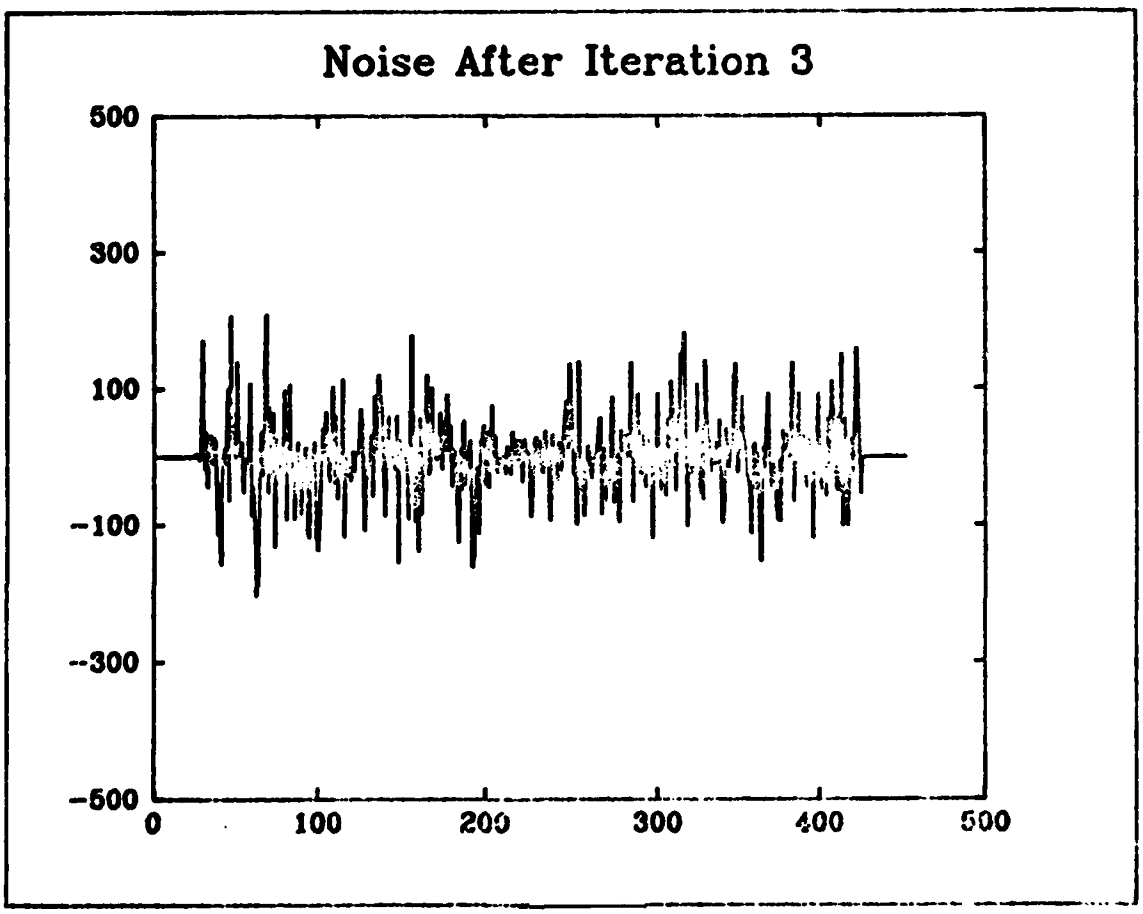

Phantom After Iteration 1
4000
3000
2000
1000
0
0
100
200
300
400
500

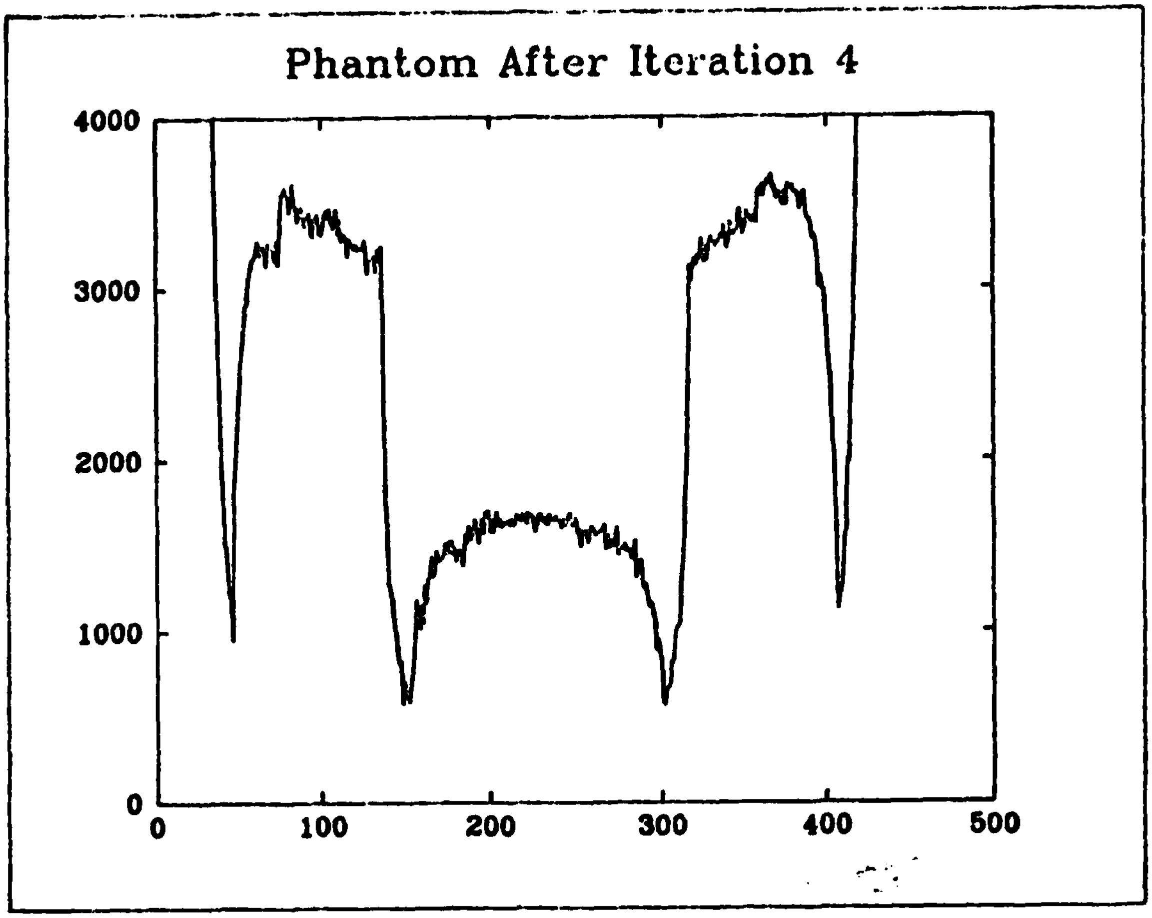

Noise After Iteration 1
500
300
100
−100
−300
−500
0
100
200
300
400
500

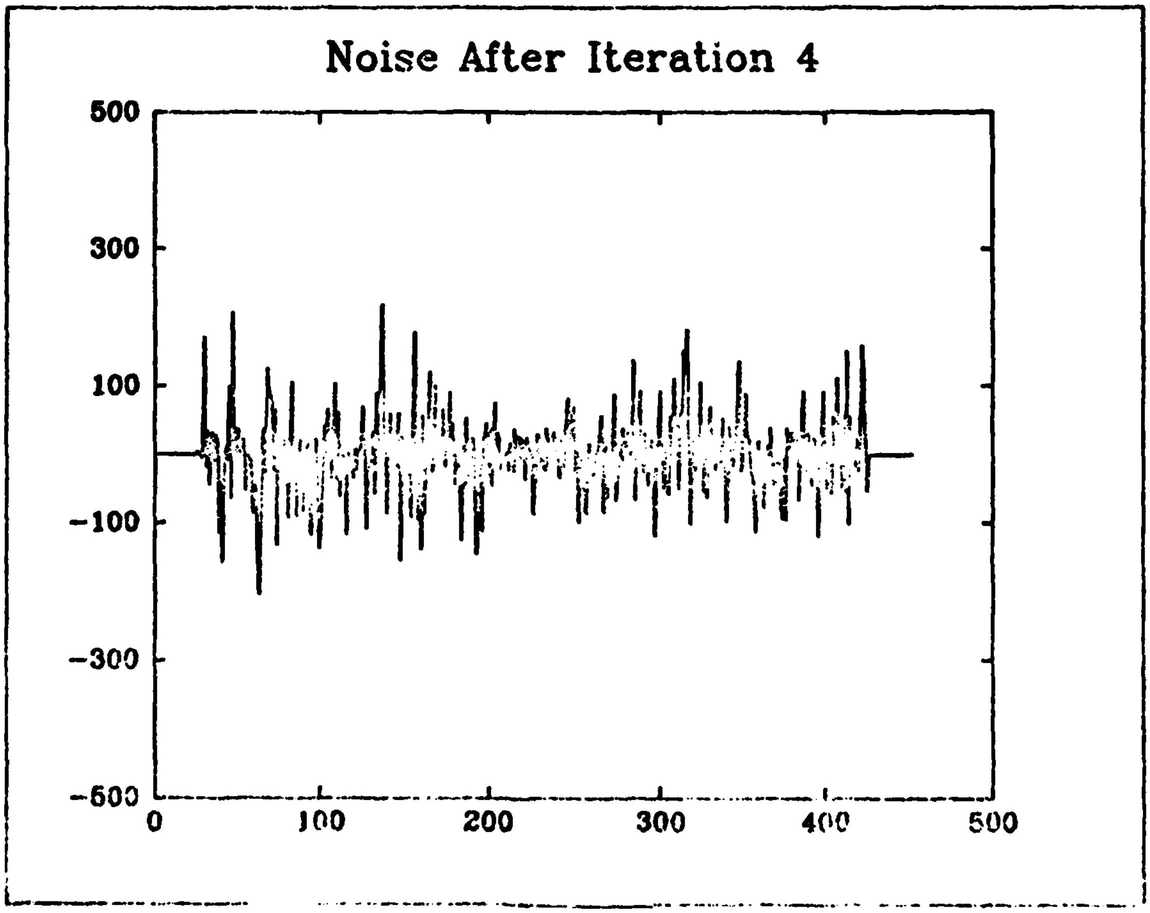

Phantom After Iteration 2
4000
3000
2000
1000
0
0
100
200
300
400
500

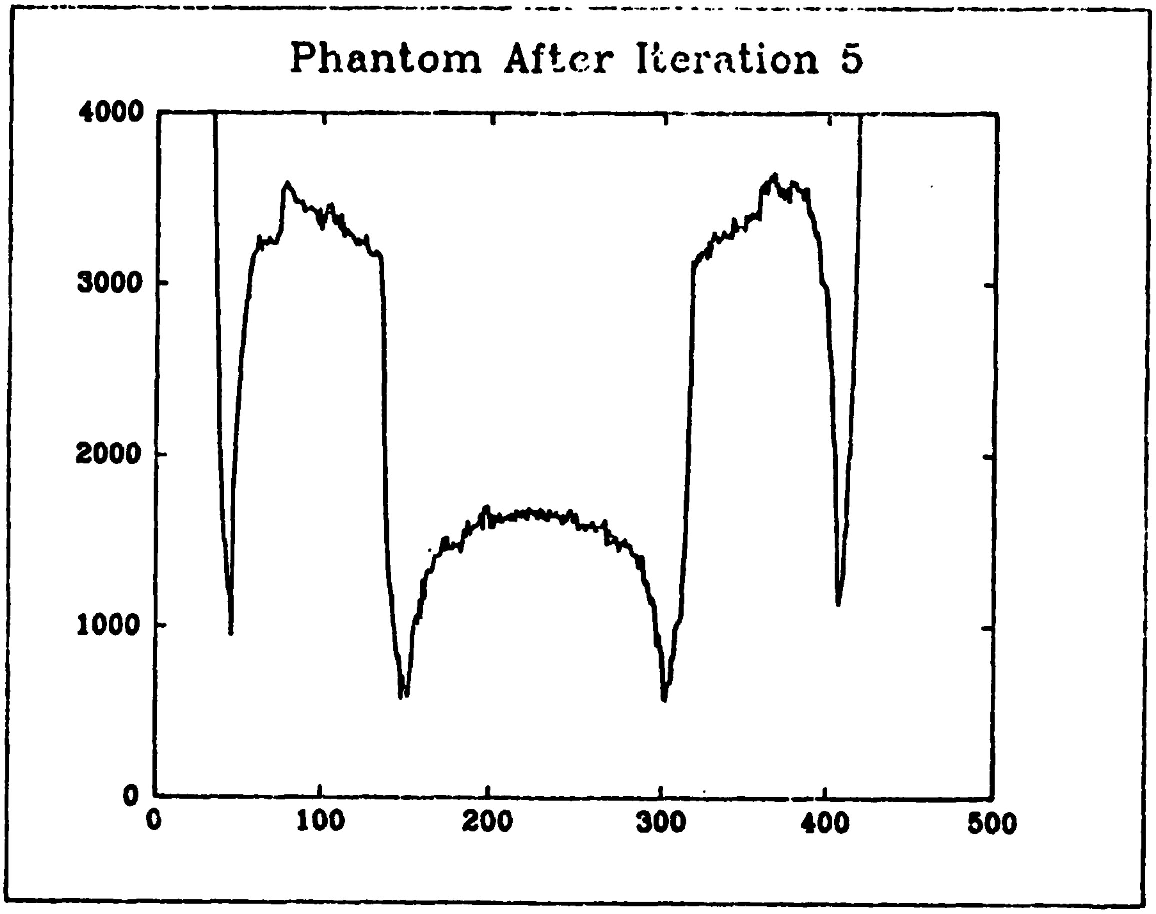

Noise After Iteration 2
500
300
100
−100
−300
−500
0
100
200
300
400
500

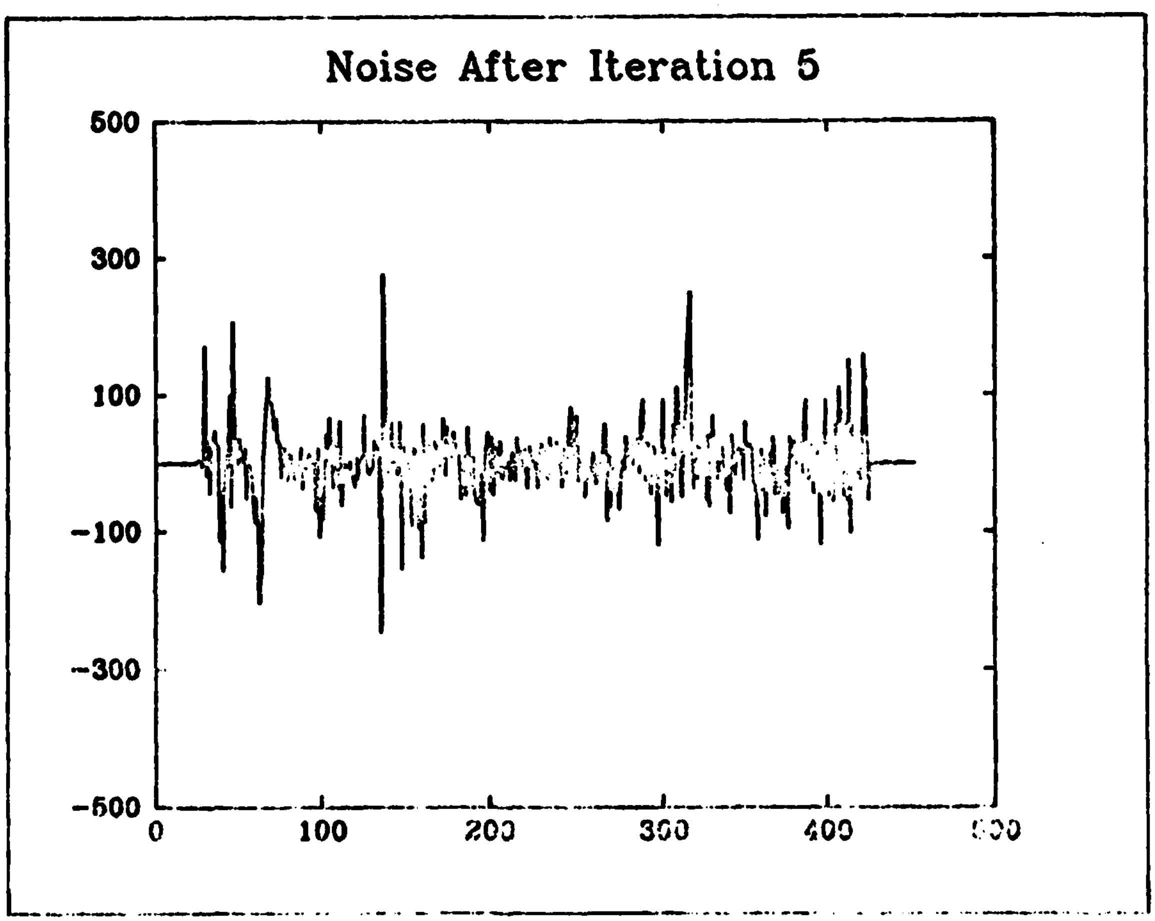

Phantom After Iteration 3
4000
3000
2000
1000
0
0
100
200
300
400
500

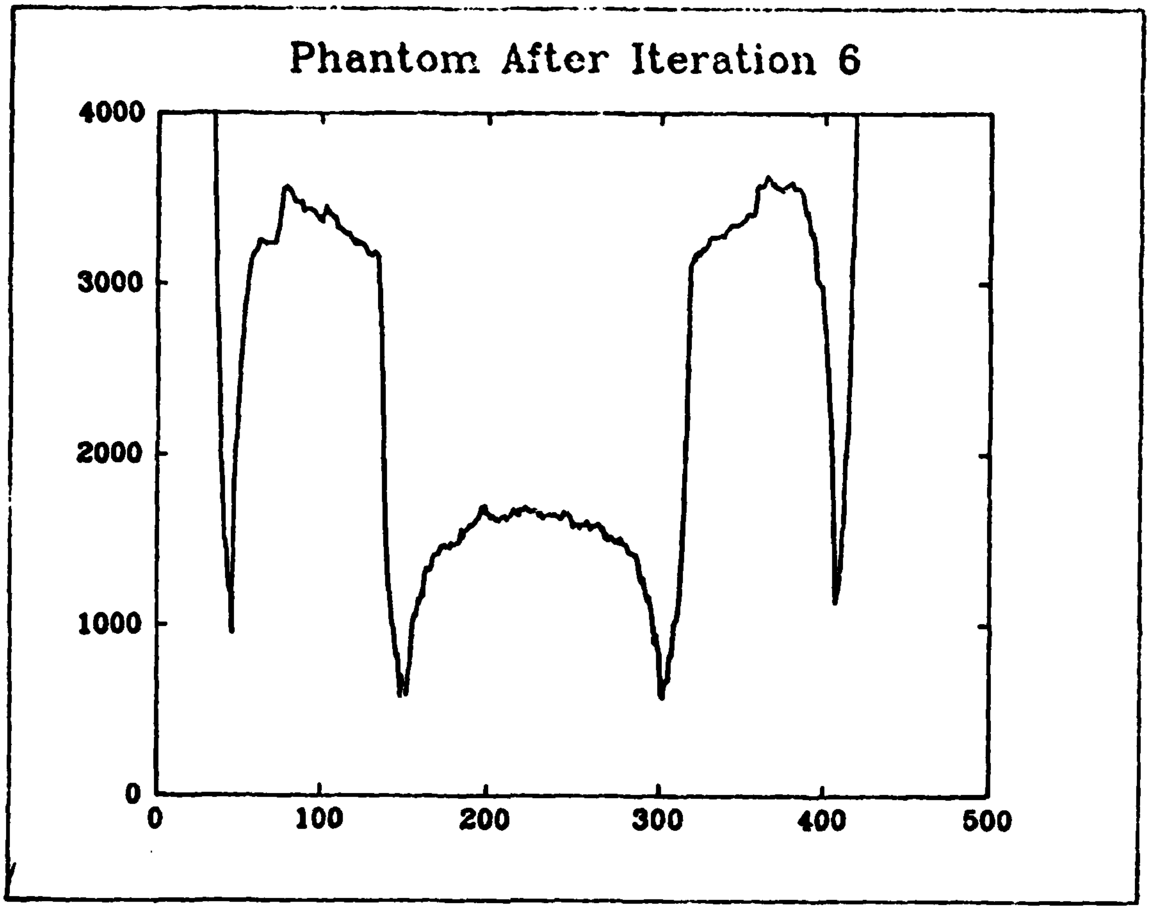

Noise After Iteration 3
500
300
100
-100
--300
-500
0
100
200
300
400
500


Phantom After Iteration 4
4000
3000
2000
1000
0
0
100
200
300
400
500

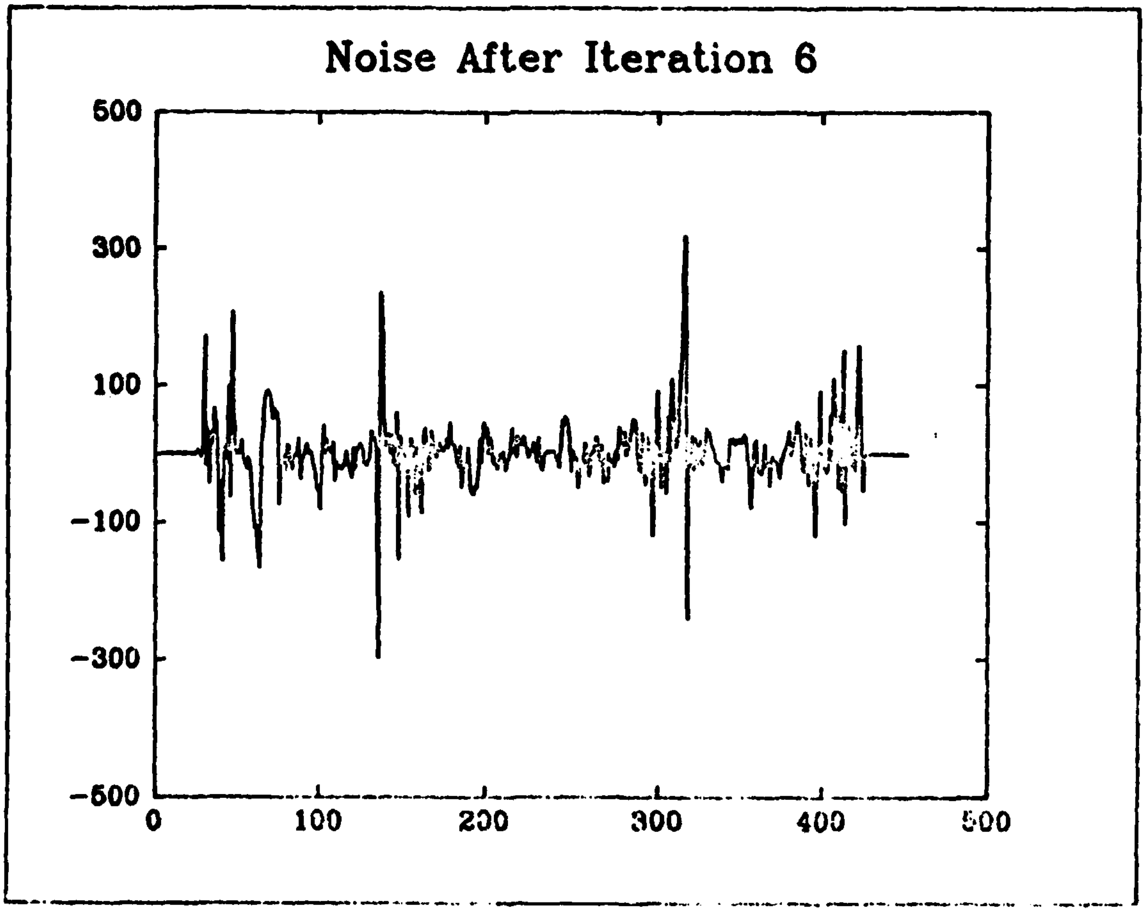

Noise After Iteration 4
500
300
100
-100
-300
-500
0
100
200
300
400
500


Phantom After Iteration 5
4000
3000
2000
1000
0
0
100
200
300
400
500

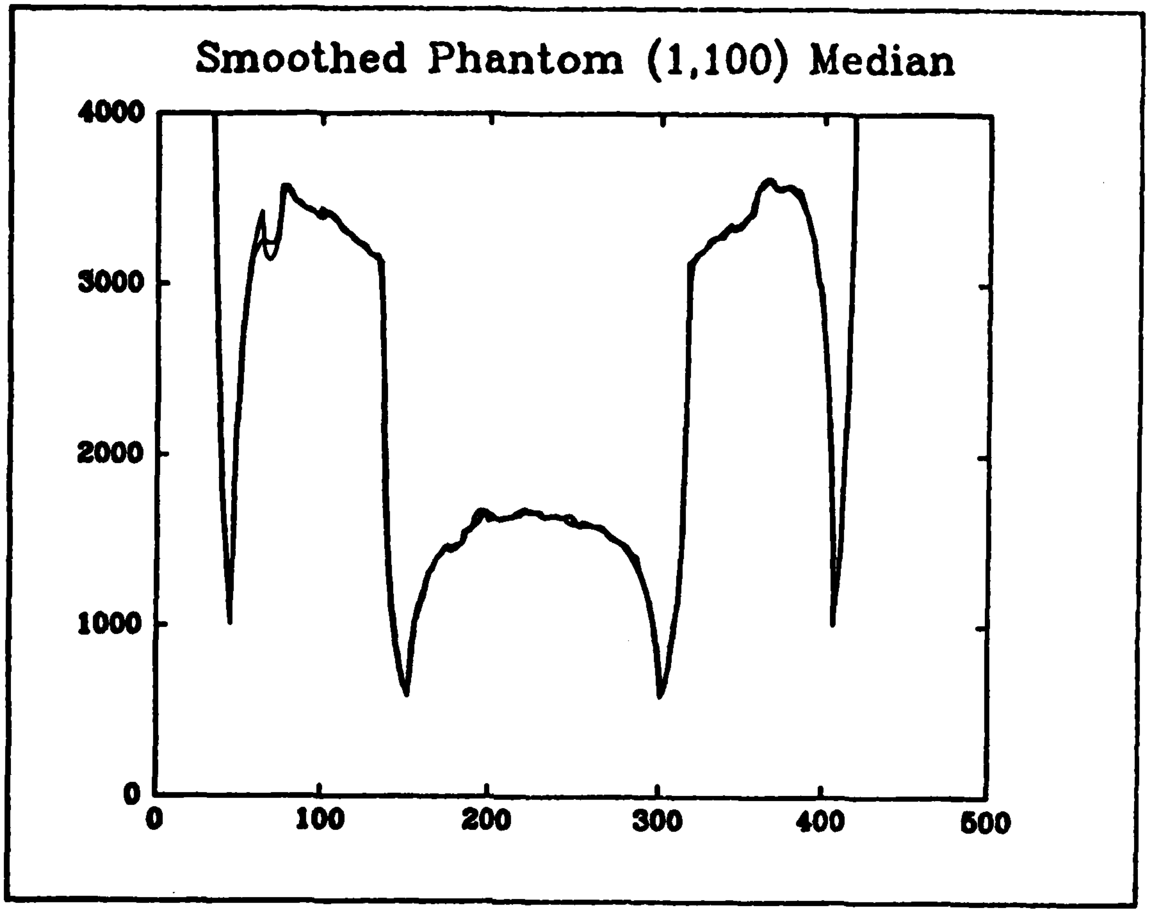

Noise After Iteration 5
500
300
100
-100
-300
-500
0
100
200
300
400



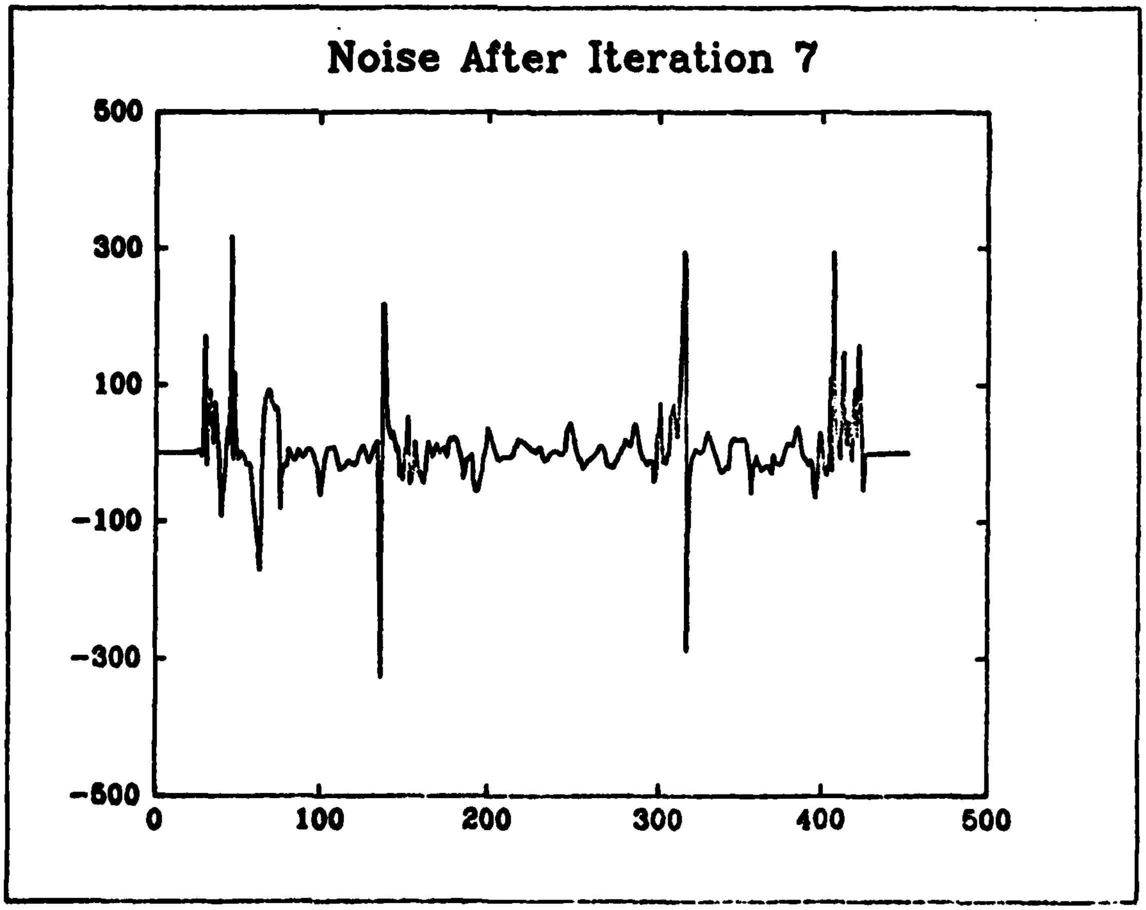
Phantom After Iteration 6
4000
3000
2000
1000
0
0
100
200
300
400
500



Noise After Iteration 6
500
300
100
-100
-300
-500
0
100
200
300
400
500



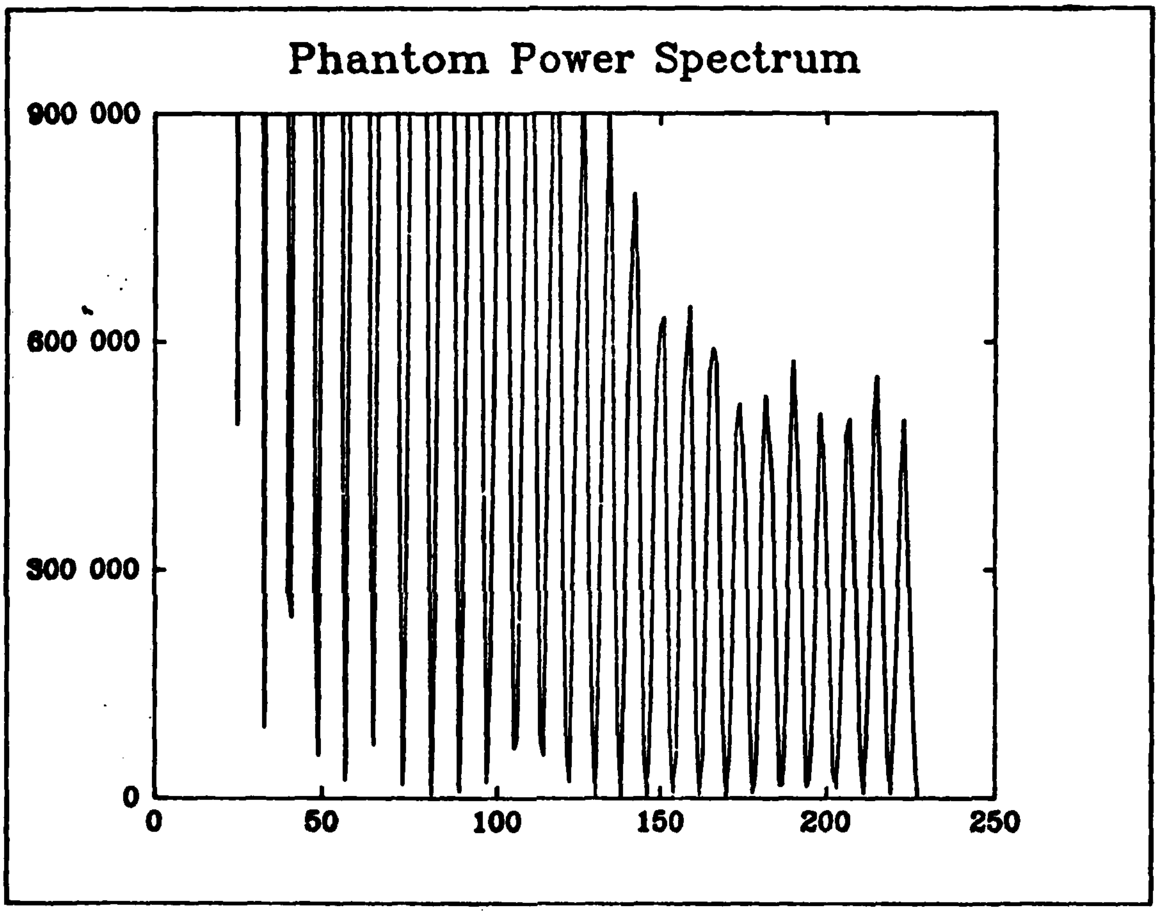
Smoothed Phantom (1,100) Median
4000
3000
2000
1000
0
0
100
200
300
400
500

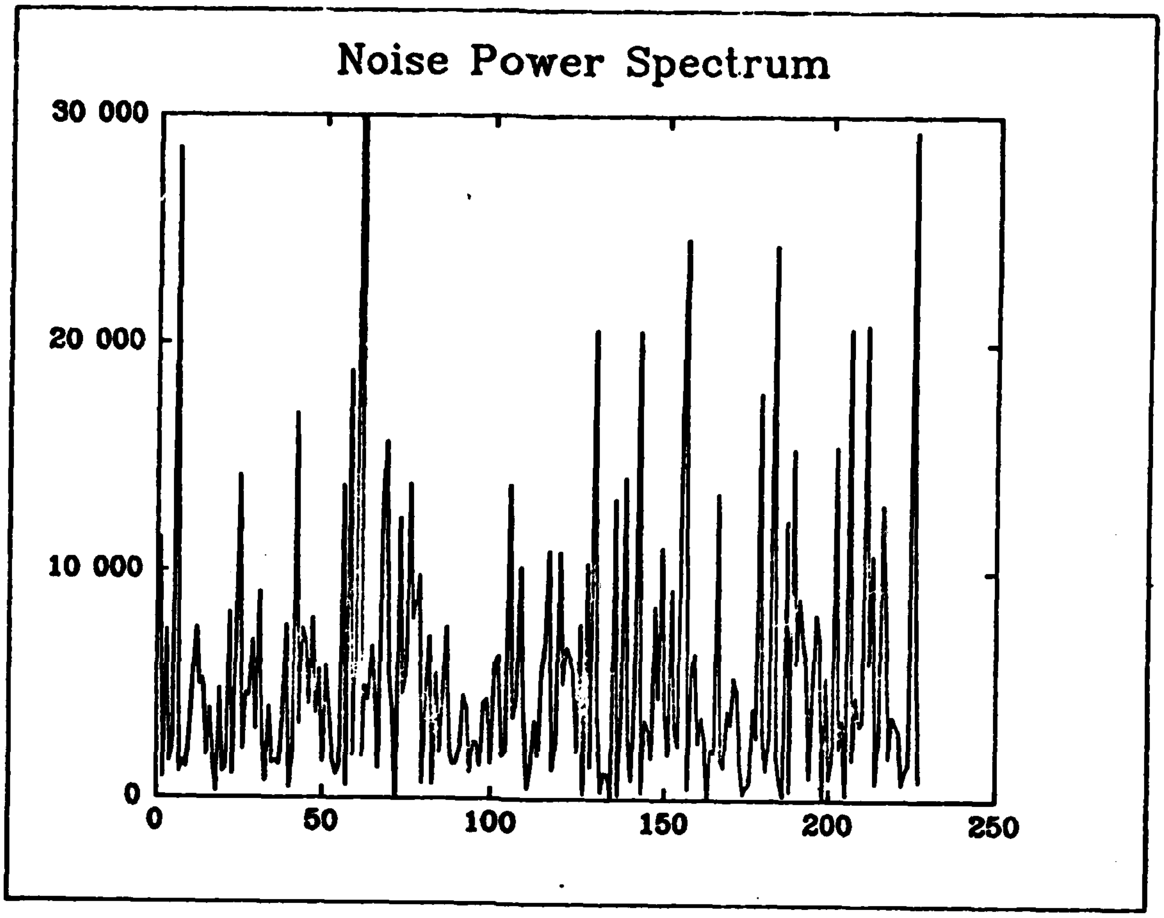


Noise After Iteration 7
500
300
100
−100
−300
−500
0
100
200
300
400
500

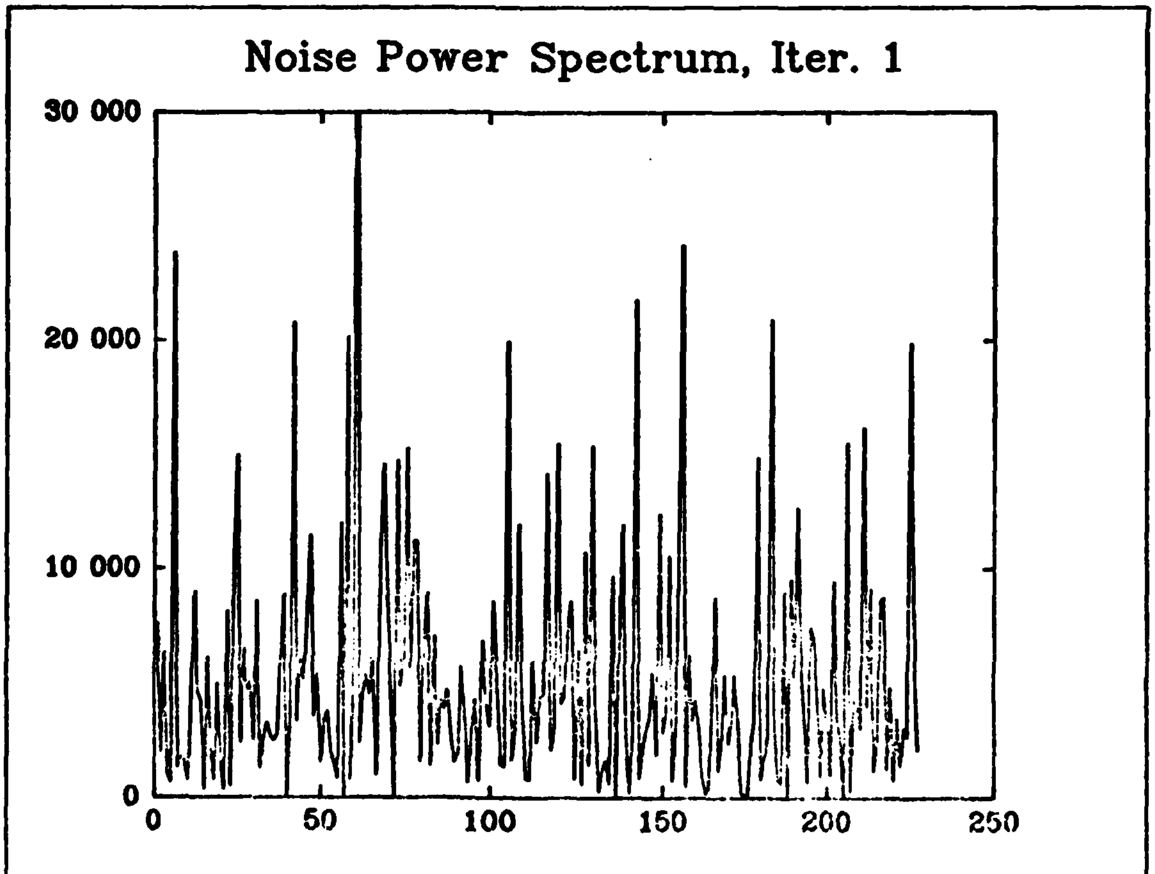
7. Power spectra.

This sections contains graphs of the power spectra (the square of the absolute value of the Fourier transform) of the noise at each step, the noise being that of the average-median process of section 5. Because of the sharp peaks the power spectrum of the phantom itself is quite wild, so successive graphs would not be instructive.

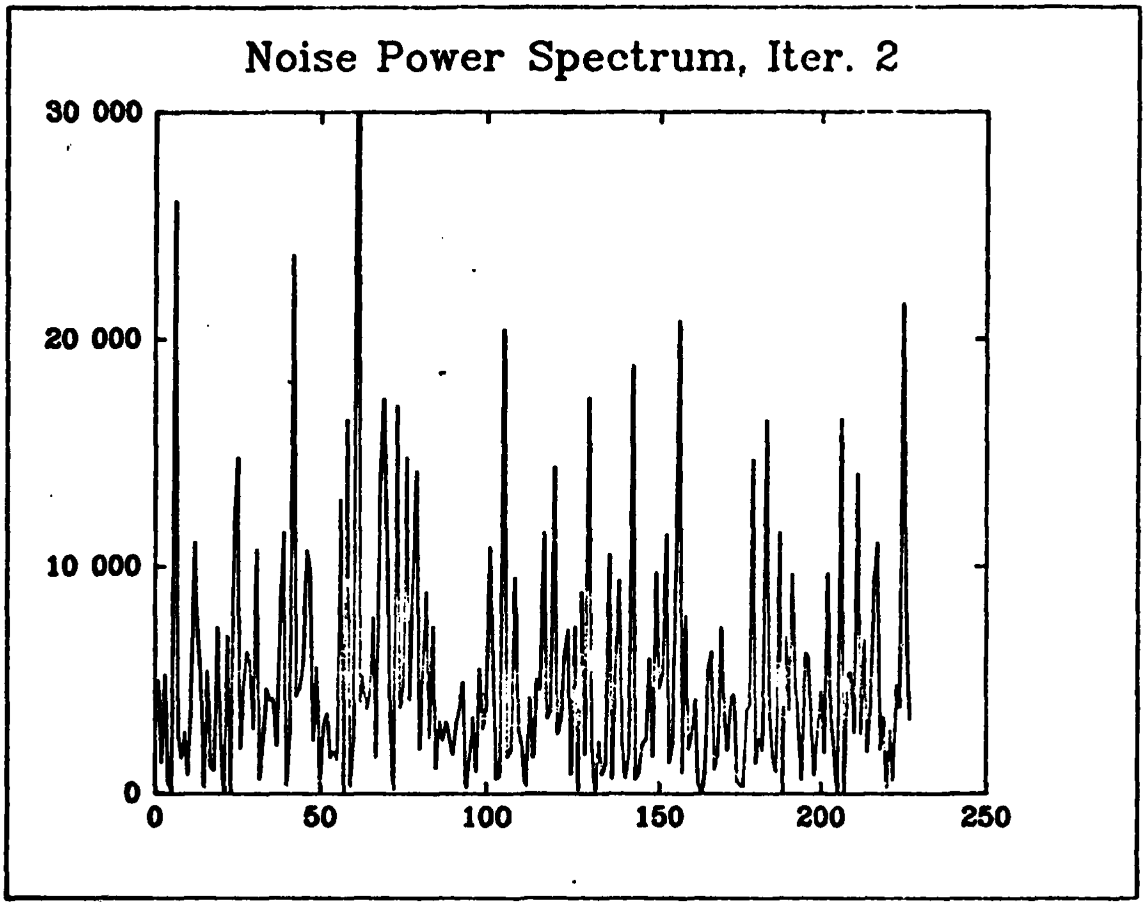


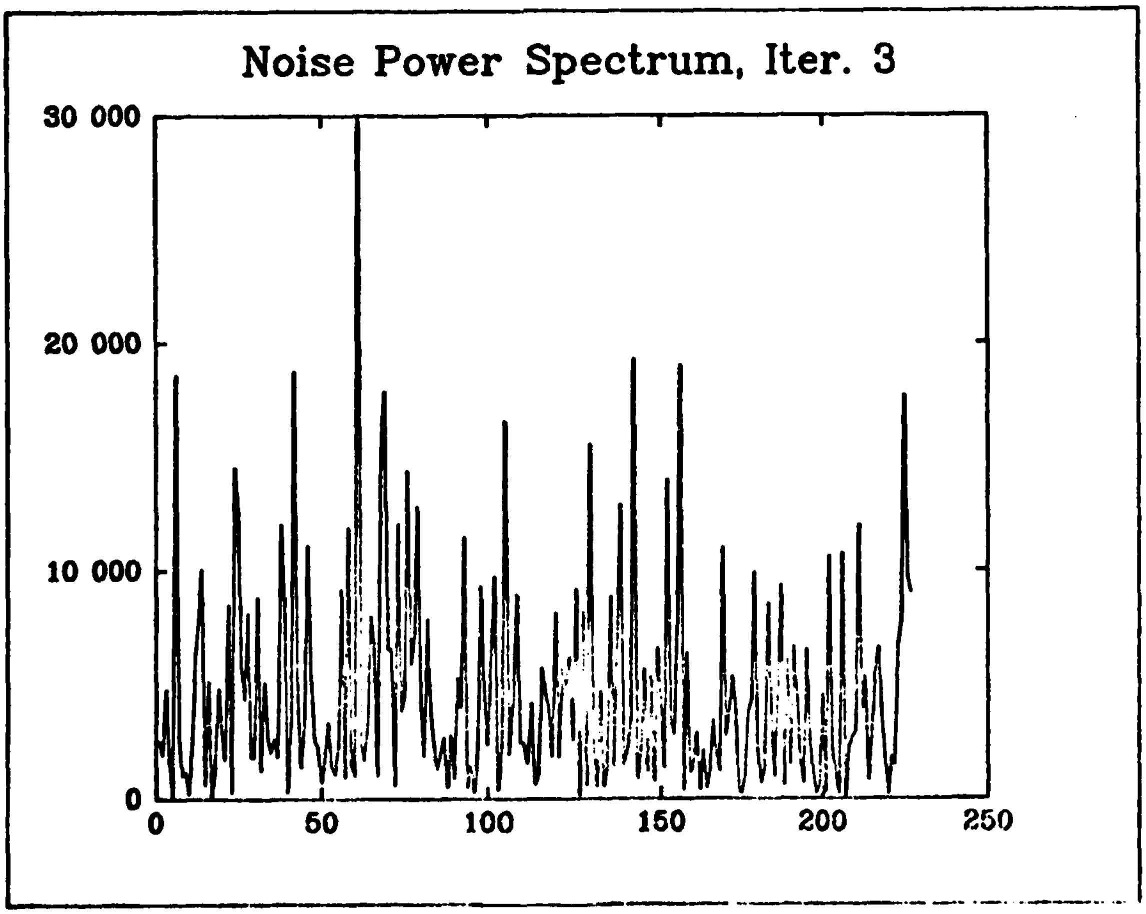


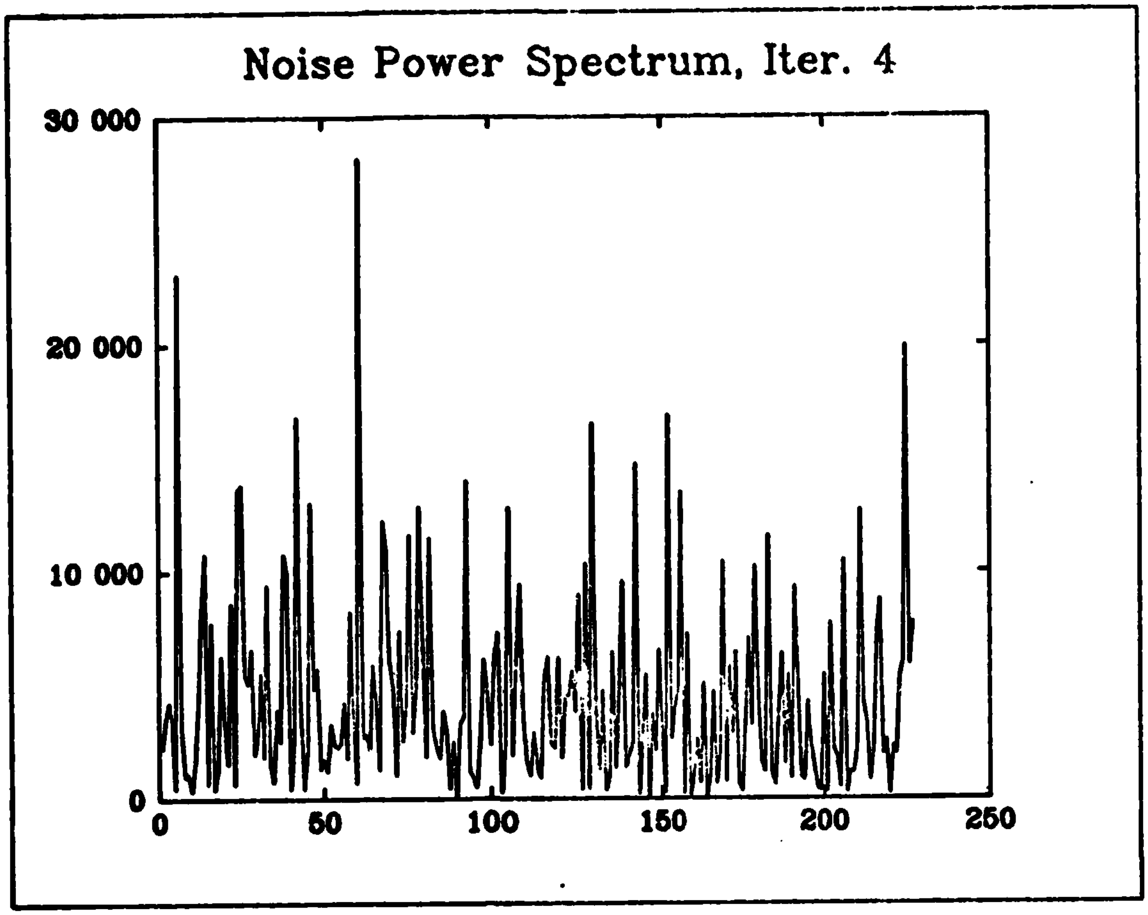
Noise Power Spectrum
30 000
20 000
10 000
0
0
50
100
150
200
250

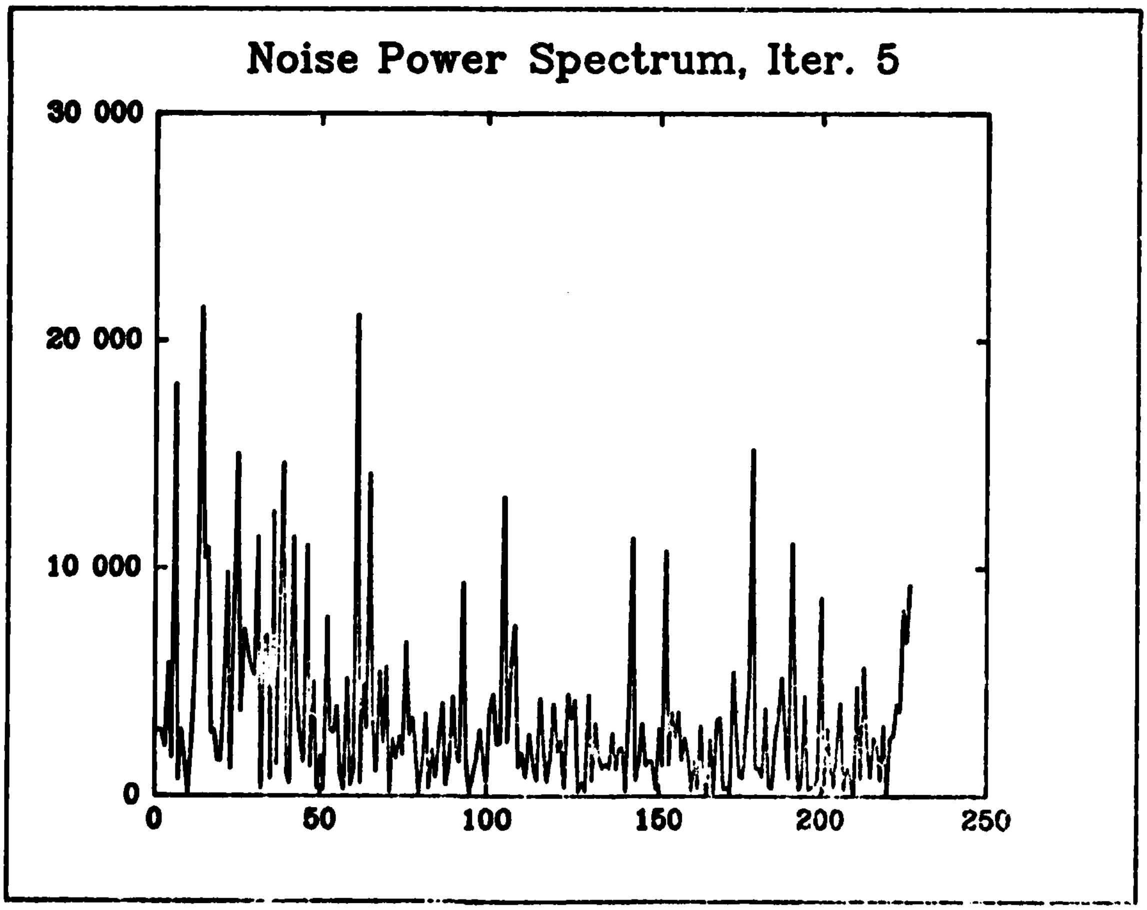


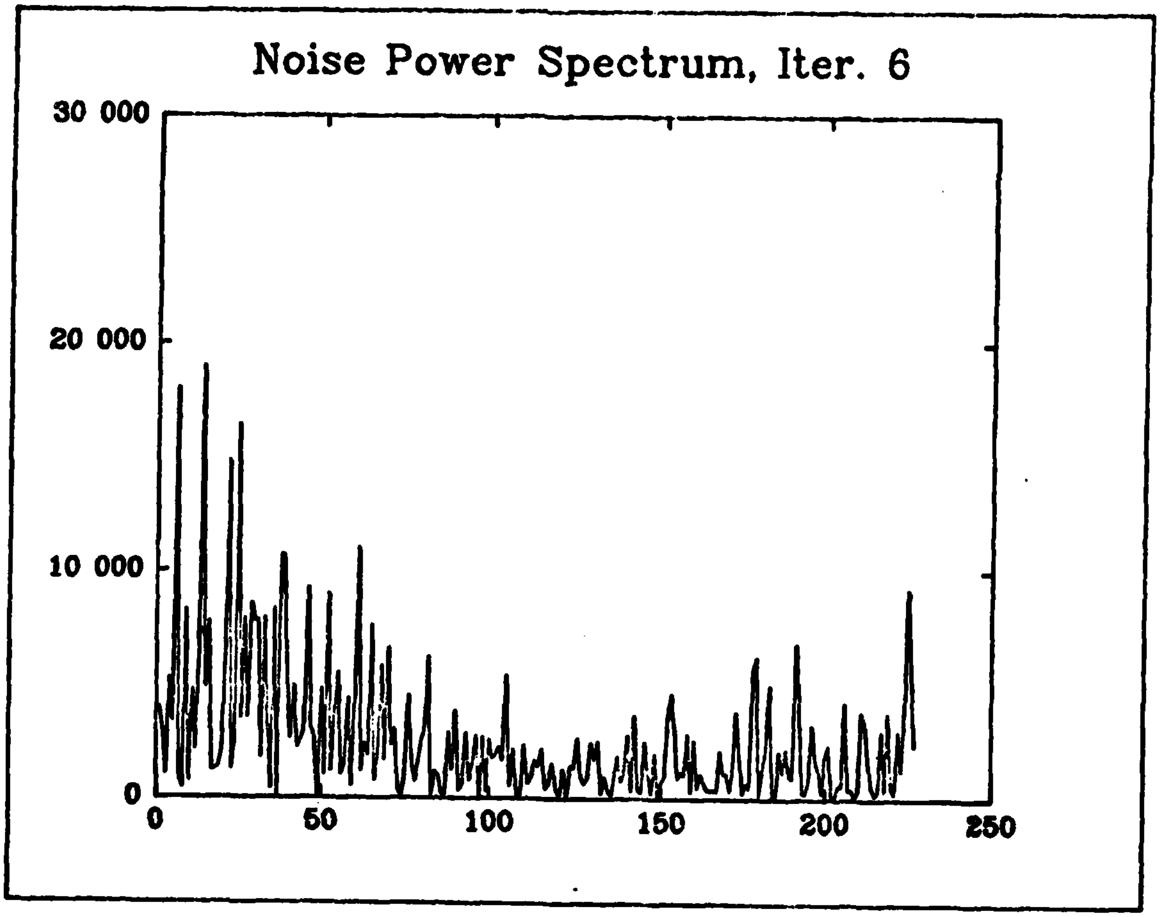
Noise Power Spectrum, Iter. 1
30 000
20 000
10 000
0
0
50
100
150
200
250

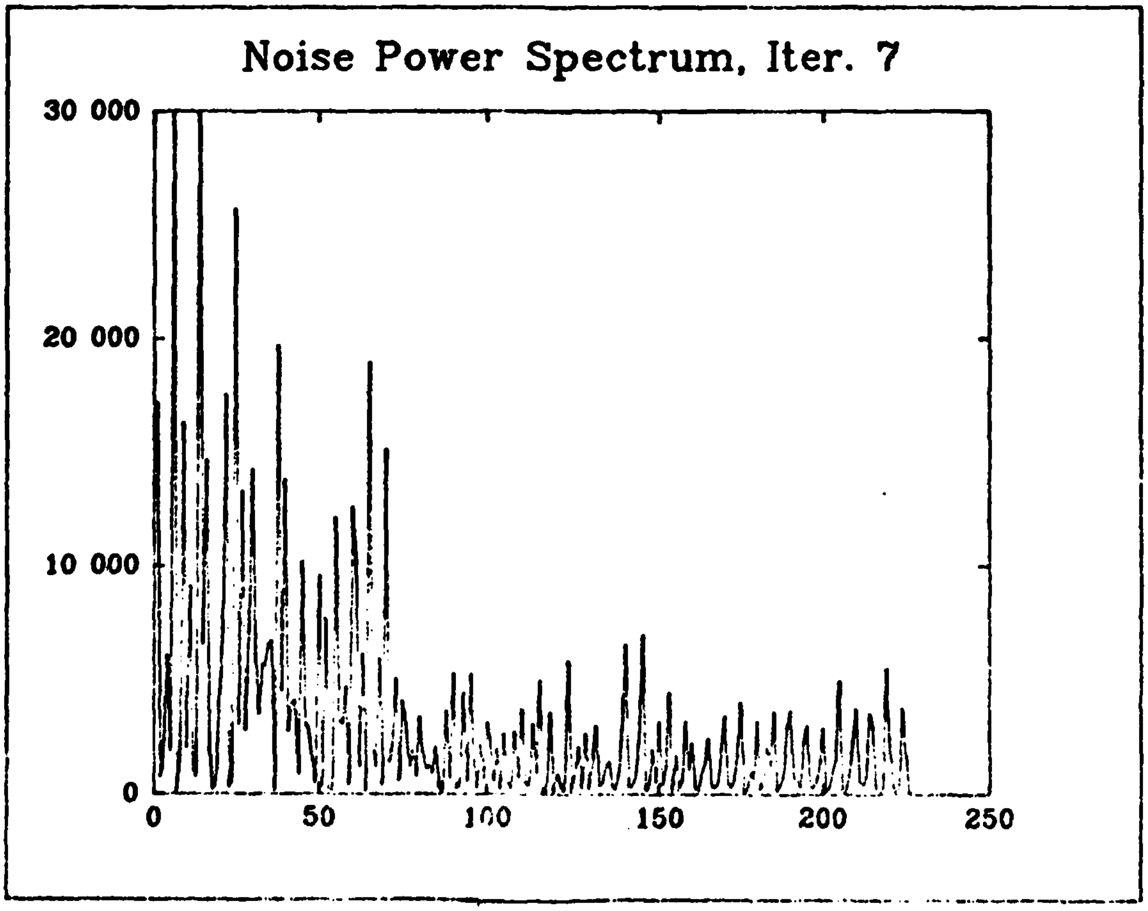

Noise Power Spectrum, Iter. 2
30 000
20 000
10 000
0
0
50
100
150
200
250


Noise Power Spectrum, Iter. 3
30 000
20 000
10 000
0
0
50
100
150
200
250


Noise Power Spectrum, Iter. 4
30 000
20 000
10 000
0
0
50
100
150
200
250


Noise Power Spectrum, Iter. 5
30 000
20 000
10 000
0
0
50
100
150
200
250


Noise Power Spectrum, Iter. 6
30 000
20 000
10 000
0
0
50
100
150
200
250


Noise Power Spectrum, Iter. 7
30 000
20 000
10 000
0
0
50
100
150
200
250

8. Noise histograms and norms.

The square of the L^2 norm of the noise at step k (i.e. the standard deviation of the phantom at step k from the true phantom) is as follows.

average-median:	5476	5319	5325	4970	4742	3660	2818	3080
median:	5476	4736	4376	3572	3136	2643	2502	2686

The number M_k used in (3.1) to identify points where corrections are made in step k+1, is obtained from a histogram H_k of the noise N_k. $H_k(j)$ is the number of points i satisfying $N_k(i) \geq j-1$. The histograms below come from the median smoothing of section 6. The number M_0, for example is computed as follows. Since p_1 = .02 and n = 452, the number of points to be corected at step 1 is .02*452 = 9. The first histogram shows that there are 9 points satisfying $N_0(i) \geq 143$, so M_0 = 143.

The first group of histograms shows the noise as it is defined by the procedure. The second group shows the true noise, i.e. the difference between the phantom at the given stage and the true phantom.

HISTOGRAM LINE 1 AFTER ITERATION 0

437	331	318	304	291	280	264	256	245	241	232	223	216	211	199
191	187	179	175	170	164	160	158	153	151	150	146	142	139	133
131	127	125	123	122	120	119	115	114	112	107	104	104	100	98
97	95	94	91	88	87	86	86	84	83	82	81	79	79	78
77	74	73	72	66	66	64	63	63	60	59	58	58	57	56
55	55	54	53	51	49	47	47	45	44	44	44	42	42	41
41	41	39	38	38	38	38	38	37	36	35	35	35	35	33
32	32	32	31	30	30	28	28	27	26	26	26	24	22	22
22	22	22	21	21	21	19	19	19	18	18	17	16	15	14
13	12	12	12	12	12	12	10	9	9	9	9	9	8	8
8	8	8	8	8	8	8	8	8	8	8	8	8	8	8
8	8	8	7	7	7	7	7	7	7	6	6	6	5	5
5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
5	5	5	5	5	5	5	4	4	4	4	4	4	3	3
3	3	3	2	2	2	2	2	2	2	2	2	2	2	2
2	2	2	2	2	2	2	2	2	1	1	1	1	1	1

HISTOGRAM LINE 1 AFTER ITERATION 1

437	329	313	301	289	278	262	254	243	239	231	221	212	206	193
188	182	177	172	167	160	156	155	150	148	147	144	141	138	130
128	124	121	118	117	114	113	109	108	106	102	100	100	96	94
92	90	89	85	82	82	82	81	79	78	76	75	73	73	72
71	68	67	66	59	59	58	57	57	53	52	51	50	49	48
47	47	46	45	43	41	39	38	36	35	35	35	34	33	32
32	32	30	29	28	28	27	27	26	25	24	24	24	24	23
22	22	22	21	20	20	18	18	18	17	17	17	15	14	14
14	13	13	12	12	12	10	10	10	9	9	8	8	7	6
5	4	4	4	4	4	4	2	2	2	2	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1										

HISTOGRAM LINE 1 AFTER ITERATION 2

441	331	321	304	291	278	266	253	242	235	228	221	212	208	199
196	188	183	179	172	166	163	157	154	151	148	138	137	134	125
120	115	111	110	104	103	100	97	93	92	88	87	87	82	81
80	79	78	78	76	75	73	73	70	69	65	65	63	60	59
57	56	56	52	50	49	48	47	47	45	44	44	44	43	41
41	40	39	39	37	35	35	35	34	34	32	32	31	30	29
28	28	26	25	24	24	24	24	23	23	23	23	23	23	22
20	19	19	17	15	15	14	14	14	12	12	12	11	11	10
9	8	8	8	8	8	6	6	6	6	6	6	6	6	6
6	6	5	5	5	5	4	4	4	4	4	3	2	2	2
2	2	2	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1						

HISTOGRAM LINE 1 AFTER ITERATION 3

441	327	310	293	280	264	252	242	227	219	211	206	198	192	183
178	171	168	163	158	152	150	146	143	140	137	131	128	125	115
110	102	96	94	88	87	84	80	77	75	72	71	69	64	64
61	60	59	59	56	55	53	53	50	49	47	45	44	42	41
37	37	37	34	33	32	31	31	31	29	28	28	27	25	23
23	22	21	21	19	17	17	17	16	16	14	14	13	12	12
12	12	10	9	8	8	8	8	7	7	7	7	7	6	5
3	2	2	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1								

HISTOGRAM LINE 1 AFTER ITERATION 4

445	320	296	282	265	251	232	221	212	204	193	191	180	170	162
151	146	141	141	135	131	125	123	118	114	111	107	103	100	96
91	85	83	81	78	76	71	68	65	63	58	55	55	49	49
48	46	44	44	41	41	41	40	38	38	38	37	37	34	31
28	27	27	26	24	24	22	21	20	19	17	16	15	15	12
12	11	7	6	4	4	3	3	2	2	2	2	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1														

HISTOGRAM LINE 1 AFTER ITERATION 5

447	327	293	273	255	239	219	207	196	186	177	168	159	145	139
130	128	120	114	107	99	92	85	78	72	64	61	50	47	41
38	34	32	28	27	25	24	21	21	20	17	15	13	10	10
8	8	7	7	7	5	4	2	2	2	2	2	2	2	2
2	2	2	2	2	2	2	2	2	2	2	1	1	1	

HISTOGRAM LINE 1 AFTER ITERATION 6

447	307	265	236	194	170	137	121	111	95	81	73	65	49	46
41	37	31	24	19	18	16	13	11	11	10	9	8	8	8
8	7	5	4	4	4	4	4	4	4	3	3	3	2	2
2	2	2	2	1										

HISTOGRAM LINE 1 AFTER ITERATION 7

449	268	174	121	85	71	61	54	49	44	41	38	33	30	29
25	25	24	22	21	19	18	17	16	16	15	15	15	15	15
14	14	13	13	13	13	12	12	12	12	11	10	8	7	7
5	4	4	4	2	1	1	1							

Histograms of True Noise

HISTORGAM AFTER ITERATION 0

452	391	380	376	371	366	358	351	347	339	330	326	318	314	307
300	295	289	281	275	273	268	259	257	248	243	235	228	220	218
216	208	208	205	202	198	197	196	192	189	186	181	180	177	174
173	172	168	166	166	164	160	157	157	155	152	149	147	145	141
138	137	136	135	134	132	130	129	124	123	123	122	120	120	119
118	117	116	114	114	113	113	113	112	111	110	109	109	108	105
104	103	101	99	97	91	90	87	87	86	85	84	83	83	78
77	77	77	74	74	71	71	71	68	67	66	64	64	63	62
61	58	56	55	55	55	55	54	53	53	52	52	52	52	51
50	50	50	47	44	42	41	40	39	38	37	36	34	32	32
32	32	32	30	30	29	27	26	26	25	25	23	23	22	21
19	19	18	18	18	18	18	18	17	16	16	16	16	16	15
14	14	14	12	12	12	12	12	11	11	11	10	10	10	10
10	10	10	10	10	10	9	9	9	9	8	8	8	7	6
6	5	5	5	5	5	5	3	3	3	3	3	3	3	3
3	3	3	3	3	3	3	3	3	3	3	3	3	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	

HISTORGAM AFTER ITERATION 1

452	391	380	376	371	366	358	351	347	339	330	326	318	314	306
299	293	286	278	272	270	265	255	253	244	239	231	224	216	214
212	204	203	200	197	193	192	191	187	184	181	176	175	172	169
168	167	163	161	161	158	154	151	151	149	146	143	141	139	134
131	130	129	128	127	125	123	122	117	116	116	115	113	113	112
111	110	109	107	107	106	106	106	105	104	103	102	102	101	98
97	96	94	92	90	84	83	80	80	79	78	77	76	76	71
70	70	70	67	67	64	64	64	61	60	59	57	57	56	55
54	51	49	48	48	48	48	47	46	46	45	44	44	44	43
43	43	43	40	37	35	34	33	32	31	30	29	27	25	25
25	25	25	23	23	22	20	19	19	18	18	16	16	15	14
12	12	11	11	11	11	11	11	10	9	9	9	9	9	9
8	8	8	6	6	6	6	6	5	5	5	5	5	5	5
5	5	5	5	5	5	4	4	4	4	3	3	3	3	2
2	1	1	1	1	1	1								

HISTORGAM AFTER ITERATION 2

452	391	380	376	371	366	356	347	343	336	326	322	314	310	302
295	289	282	273	267	265	260	250	248	239	234	226	219	211	209
207	199	198	195	192	188	187	186	182	179	176	171	170	167	164
163	162	158	156	156	153	149	146	146	144	141	138	136	134	129
126	125	124	123	122	120	118	117	111	110	110	109	107	107	106
105	104	103	101	101	100	100	100	99	98	97	96	96	94	91
90	89	87	85	83	77	76	73	73	72	71	70	69	69	64
63	63	63	60	60	57	57	57	54	53	52	50	50	49	48
47	44	42	41	41	41	41	40	39	39	38	37	37	37	36
36	36	36	33	30	28	27	26	25	25	24	23	22	21	21
21	21	21	19	19	18	17	16	16	15	15	13	13	13	12
11	11	11	10	10	10	10	10	9	8	8	8	8	8	8
7	7	7	5	5	5	5	5	5	5	5	5	5	5	5
5	5	5	5	5	5	4	4	4	4	3	3	3	3	2
2	1	1	1	1	1	1								

HISTORGAM AFTER ITERATION 3

452	390	379	374	369	363	353	342	338	331	321	315	307	302	294
287	281	273	264	257	255	250	240	238	228	223	214	206	198	196
194	186	185	182	179	175	174	173	169	166	163	158	157	153	150
149	148	144	142	142	140	136	133	133	131	128	125	124	121	116
113	112	111	110	109	105	103	102	96	95	95	94	92	92	91
90	89	88	86	86	85	85	85	84	83	82	81	81	79	76
75	74	72	70	68	62	61	58	58	57	56	55	54	54	49
48	48	48	45	45	42	42	42	39	38	37	35	35	34	33
33	30	28	28	28	28	28	27	26	26	26	25	25	25	24
24	24	24	22	19	18	17	16	15	15	15	15	14	14	14
14	14	14	12	12	11	10	10	10	9	9	8	8	8	7
7	7	7	6	6	6	6	6	5	5	5	5	5	5	5
4	4	4	3	3	3	3	3	3	3	3	3	3	3	3
3	3	3	3	3	3	3	3	3	3	2	2	2	2	1
1														

HISTORGAM AFTER ITERATION 4

452	390	378	373	368	362	351	339	335	328	317	311	303	298	290
283	277	269	259	252	250	244	234	232	222	216	207	198	190	188
186	178	177	174	171	167	166	165	161	156	153	147	146	142	139
138	137	133	130	130	128	123	120	120	118	116	113	112	108	103
100	99	98	97	96	92	90	89	83	82	82	81	79	79	78
77	76	75	73	73	72	72	72	71	70	69	68	68	65	62
61	60	58	56	54	50	49	47	47	46	46	45	44	44	40
39	39	39	36	36	34	34	34	31	30	29	28	28	27	26
26	23	21	21	21	21	21	20	19	19	19	18	18	18	17
17	17	17	16	14	13	13	13	13	13	13	13	12	12	12
12	12	12	10	10	10	9	9	9	8	8	7	7	7	7
7	7	7	6	6	6	6	6	5	5	5	5	5	5	5
4	4	4	3	3	3	3	3	3	3	3	3	3	3	3
3	3	3	3	3	3	3	3	3	3	2	2	2	2	1
1	1	1	1	1	1	1	1	1	1					

HISTORGAM AFTER ITERATION 5

452	390	377	372	364	359	347	334	330	323	311	304	296	291	280
273	264	255	244	236	233	225	212	209	198	192	185	176	168	167
165	157	154	150	147	141	140	137	133	127	124	118	117	113	110
110	109	105	100	99	97	91	88	88	86	84	82	81	77	73
70	68	66	65	64	61	59	58	54	53	53	51	49	49	48
47	46	46	44	44	43	43	42	41	40	40	39	39	36	36
36	36	36	35	34	30	30	28	28	28	28	28	27	27	25
25	25	25	24	24	23	23	23	21	20	19	18	18	18	18
18	16	16	16	16	16	16	16	15	15	15	15	15	15	15
15	15	15	15	15	14	14	14	14	14	13	13	13	13	13
13	13	13	11	11	11	10	10	10	9	9	8	8	8	8
8	8	8	7	7	7	7	7	6	6	6	6	6	6	6
6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
6	6	6	6	6	6	6	6	6	6	5	5	5	5	3
3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
3	3	3	3	3	3	3	3	2	2	2	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1							

HISTORGAM AFTER ITERATION 6

452	392	378	371	357	348	331	319	314	306	292	283	274	266	254
246	235	225	213	206	201	192	177	171	159	153	147	139	129	127
123	118	116	112	109	104	103	100	99	95	92	88	87	86	84
82	81	77	73	71	69	64	62	61	60	59	58	56	53	52
51	49	47	46	45	44	43	43	41	41	41	41	41	41	41
41	39	39	37	37	37	36	35	34	34	34	34	34	31	31
31	31	30	29	28	26	26	26	26	26	26	26	25	25	23
22	22	22	21	21	21	20	20	18	18	18	17	17	17	17
17	15	15	15	15	15	15	15	15	15	15	15	15	15	15
15	15	15	15	15	15	15	14	14	14	14	14	14	14	14
14	14	14	12	12	12	11	11	11	10	10	9	9	9	9
9	9	9	8	8	8	8	8	7	7	7	7	7	7	7
7	7	7	7	7	7	7	7	7	7	7	6	6	6	6
6	6	6	6	6	6	6	6	6	6	6	6	6	6	5
5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
5	5	5	5	5	5	5	5	5	5	5	5	4	4	4
4	4	3	3	3	3	3	3	3	3	3	3	3	3	3
3	3	3	2	2	2	2	2	2	2	2	2	2	2	2
2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
2	2	2	2	2	2	2	2	2	2	2	2	2	2	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1									

HISTORGAM AFTER ITERATION 7

452	393	370	356	345	340	327	307	293	283	270	258	240	231	221
214	205	191	177	162	154	147	139	129	121	118	116	113	110	105
104	103	97	93	91	90	87	83	81	79	75	74	70	68	67
66	64	64	63	62	61	60	59	58	58	54	51	51	49	47
45	44	42	41	41	41	41	40	40	39	38	37	36	35	34
33	33	32	32	32	31	31	31	31	30	29	29	29	28	28
28	26	26	25	24	22	21	21	21	21	21	21	21	21	21
20	20	20	19	19	18	17	16	16	16	16	16	16	16	16
15	15	14	14	14	14	14	14	14	14	14	14	14	14	14
14	14	14	14	14	14	14	14	14	13	13	13	13	13	12
12	12	12	12	12	12	12	12	12	12	12	11	11	11	11
11	11	11	11	11	11	11	11	9	9	9	9	9	9	9
9	9	9	9	9	9	9	9	9	9	9	9	9	9	9
8	8	8	8	8	8	8	8	8	8	8	8	8	8	8
8	8	8	8	8	8	8	7	7	7	7	6	6	6	6
6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
6	6	6	6	6	6	4	4	4	4	4	4	2	2	2
2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
2	2	2	2	1	1	1	1	1	1	1	1	1	1	1

9. Remarks.

During the past few years variants of these iterative noise elimination procedures have been used with several kinds of data: various medical radiographs, neutron radiographs of nuclear fuel assemblies, ultrasound CT breast scans, CT x-ray reconstructions, and CT x-ray scans. Each kind of data has individual features and special difficulties, and must be treated in terms of these features and difficulties and the use to which the data is to be put.

Dominant features of the present data are the large, sharp steel spikes and the large drop at the edges, features that required the cutoffs described in section 3. To some extent, probably to a large extent, these features are responsible for the superiority of the median smoothing over the others. Also, they are responsible for the apparent increase in the noise at the last stage of the procedure, which comes from errors at a half dozen points of very large slope. It is not clear whether anything should or can be done about this. The test would be to use the battery data in CT reconstructions. This has not been possible yet, but will be done in the near future. (In their role in the CT reconstruction formulas, the systematic errors at the peaks could be much more dangerous than random noise.)

References

1. K.T. Smith, D.C. Solmon, and S.L. Wagner. Practical and mathematical aspects of reconstructing objects from radiographs. BAMS 1977 1227-1270.

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER 2464	2. GOVT ACCESSION NO. AD-A127 756	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) ITERATIVE NOISE ELIMINATION Preliminary Report		5. TYPE OF REPORT & PERIOD COVERED Summary Report - no specific reporting period
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) K. T. Smith		8. CONTRACT OR GRANT NUMBER(s) DAAG29-80-C-0041 & MCS-8101586
9. PERFORMING ORGANIZATION NAME AND ADDRESS Mathematics Research Center, University of Wisconsin 610 Walnut Street Madison, Wisconsin 53706		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS Work Unit Number - 1 - Applied Analaysis
11. CONTROLLING OFFICE NAME AND ADDRESS * see below		12. REPORT DATE January, 1983
		13. NUMBER OF PAGES 47
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES *U.S. Army Research Office P.O. Box 12211 Research Triangle Park North Carolina 27709	National Science Foundation Washington, D.C. 20550	Tektronix Co. Beaverton, OR 97005
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Noise, Computed tomography		

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A common feature of the empirical data in applied mathematics is a low signal to noise ratio. Defined loosely as the ratio of the magnitude of essential signals to the magnitude of the worst noise, it often runs between 1/10 and 1/15, as was the case in the early work on computed tomography with digitized medical x-ray films which was the origin of this project. In such situations, straight-forward linear smoothing procedures tend to broaden the noise peaks without reducing the magnitude sufficiently.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

20. Abstract (cont.)

A common redeeming feature of the noise is that the largest peaks are quite sparse, lesser peaks are more frequent but still sparse, etc. This suggests a stepwise smoothing in which only the worst peaks are removed during the early steps when there is no very good way to remove them.

Such a procedure, devised for use with medical x-ray films, is described in [1]. In the meantime it has been modified in various ways and used with other data from several sources. A detailed account of experiments with the procedure will be given elsewhere. The purpose of the present report is to describe the procedure and to show the results of a series of tests with data from a computed tomography x-ray scan of a defective battery. The scan was taken with a developmental version of an industrial scanner at the General Electric Research and Development Center by H. J. Scudder.

END

FILMED

6-83

DTIC